一百學年度第一學期微積分會考試題 (A 卷)

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

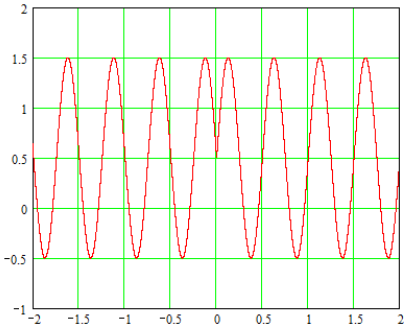
(2) 測驗時間 110 分鐘。試卷加答案卷、答案卡共計 7 頁。

(3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. The graph below comes from an experimental data. What is the best model function to describe the data ?



(A) $\cos(4\pi x)+0.5$.

(B) $\sin(4\pi x)+0.5$.

(C) $\sin(4\pi|x|)+0.5$.

(D) $\cos(4\pi|x|)+0.5$.

2. Let $A=\{0.6,0.7,0.8,0.9\}$.

Find the **largest** number, δ, in A such that

$|\sqrt{4x+5}-3|<0.6$, whenever $|x-1|<\delta$.

(A) $\delta=0.6$, (B) $\delta=0.7$, (C) $\delta=0.8$, (D) $\delta=0.9$.

3. $\lim_{x\to 0} x^2\cos\left(\frac{3}{x^4}\right)$

(A) 0, (B) 1, (C) 3, (D) ∞.

4. Let $f\left(\frac{1+x}{1-x}\right)=x$. Find $f'(2)$.

(A) 1; (B) $\frac{1}{3}$; (C) $\frac{1}{9}$; (D) $\frac{2}{9}$.

5. $\lim_{x\to\infty}\left(\frac{\ln x}{x^e}\frac{x^\pi}{e^x}\right)=$

(A) ∞, (B) π, (C) $\frac{e}{\pi}$, (D) 0.

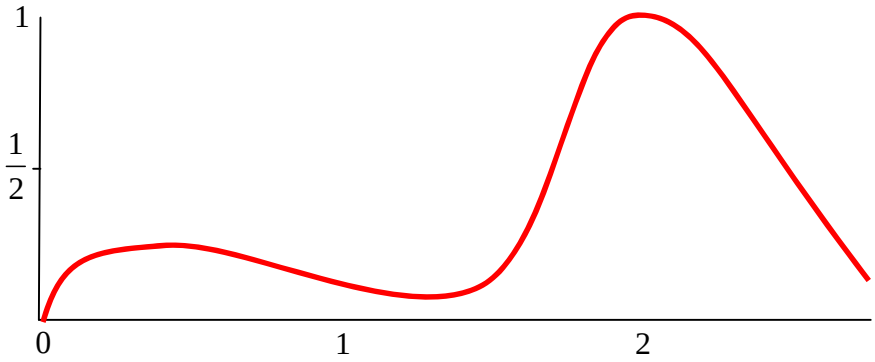
6. $\int_0^{\frac{1}{2}}\frac{x^2}{\sqrt{1-x^2}}dx=$

(A) $\frac{\pi}{12}$; (B) $\frac{\pi}{12}-\frac{\sqrt{3}}{8}$; (C) $\frac{\sqrt{3}}{8}-\frac{\pi}{12}$; (D) $\frac{\sqrt{3}}{8}$.

7. Find the **length** of the curve $y=\int_1^x\sqrt{t^3-1}\,dt,\ 1\le x\le 4$.

(A) $\frac{62}{5}$, (B) $\frac{64}{5}$, (C) $\frac{66}{5}$, (D) $\frac{68}{5}$.

8. The graph of a differentiable function f is shown below :



List the following integrals in **order** from the **smallest** to the **largest**

$$\text{I}=\int_0^1 f(x)dx \qquad \text{II}=\int_0^2 f(x)dx \qquad \text{III}=\int_0^1 f^2(x)dx \qquad \text{IV}=\int_0^2 f'(x)dx$$

(A) I < II < III < IV, (B) II < III < I < IV,
(C) III < I < II < IV, (D) IV < III < I < II .

9. Let $I=\lim_{x\to 0^+}\frac{1}{x}\left(\int_{1-x}^{1}\frac{1}{t^3+t}dt\right)$. Which of the following statement is **true** ?

(A) $I=-1$. (B) $I=1$. (C) $I=-\frac{1}{2}$. (D) $I=\frac{1}{2}$.

10. The **area** of **one** of the **loops** enclosed by the polar curve $r^2=9\cos 5\theta$ is

(A) $\frac{1}{10}$, (B) $\frac{9}{10}$, (C) $\frac{\pi}{3}$, (D) $\frac{9}{5}$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

11. Suppose that f'' and g'' exist. Which of the following statements are **true** ?

(A) $\left(f(x)g(x)\right)''=f(x)g''(x)+f''(x)g(x)$.

(B) $f(x)g''(x)-f''(x)g(x)=\frac{d}{dx}\left[f(x)g'(x)-f'(x)g(x)\right]$.

(C) $\frac{d^2}{dx^2}\left[f\left(g(x)\right)\right]=f''\left(g(x)\right)\left[g'(x)\right]^2+f'\left(g(x)\right)g''(x)$.

(D) $\frac{d}{dx}\left(f\left(x+g\left(\cos x\right)\right)\right)=f'\left(x+g\left(\cos x\right)\right)\left(1+g'\left(\sin x\right)\right)\cos x$.

12. Let $f(x)=\frac{x^3}{x^2+\pi}$, then

(A) f is a continuous function,
(B) f does not have any asymptote,
(C) f has only one inflection point,
(D) f does not have maximum and minimun values.

13. Let $I=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{2n}\left[4-6\left(\frac{i}{2n}\right)^2\right]$. Which of the following statements are **true** ?

(A) $I=\int_0^1\left(2-\frac{3}{4}x^2\right)dx$. (B) $I=\int_0^{\frac{1}{2}}\left(4-6x^2\right)dx$.

(C) $I=\int_0^1\left(4-6x^2\right)dx$. (D) $I=\int_0^{\frac{1}{2}}\left(2-\frac{3}{4}x^2\right)dx$.

14. Which of the following integrals are the **area** enclosed by $y = x$ and $y^2 = x + 6$?

(A) $\int_{-2}^{3} \left(y - y^2 + 6 \right) dy$;

(B) $\int_{-6}^{-2} 2\sqrt{x+6}\, dx - \int_{-2}^{3} \left(\sqrt{x+6} - x \right) dx$;

(C) $\int_{-2}^{3} \left(y - y^2 - 6 \right) dy$;

(D) $\int_{-6}^{-2} 2\sqrt{x+6}\, dx + \int_{-2}^{3} \left(\sqrt{x+6} - x \right) dx$.

15. Which of the following improper integrals are **convergent** ?

(A) $\int_1^{\infty} \frac{1}{x} dx$. (B) $\int_0^{\infty} \frac{1}{x^2+1} dx$. (C) $\int_1^{\infty} e^{-x} dx$. (D) $\int_{-\infty}^{\infty} \frac{1}{x^4 + x^2 + 1} dx$.

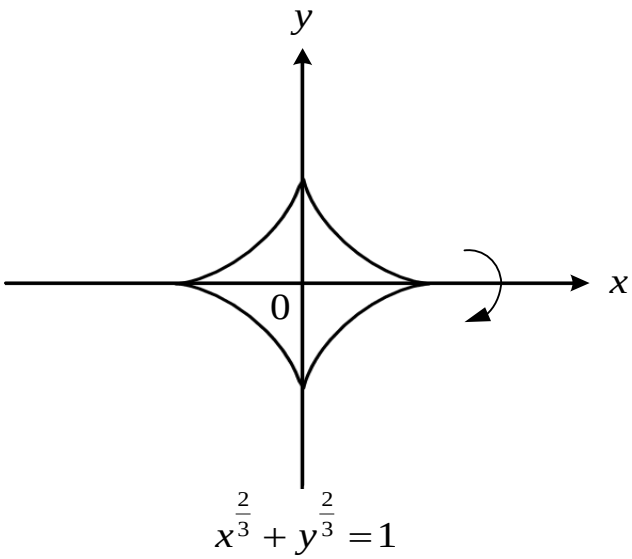
填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. Let $f(x) = \begin{cases} -x^2 + 4x - 3, & \text{if } x \le \frac{5}{2} \\ ax + b, & \text{if } x > \frac{5}{2} \end{cases}$. If f is a **differentiable** function on $\mathbb{R}$, then $(a, b) =$ ___(1)___ .

2. $\lim_{x \to \infty} \left(e^x + x \right)^{\frac{e}{x}} =$ ___(2)___ .

3. The **tangent line** equation of the parametric equation $x = \cos\theta + \sin 2\theta$, $y = \sin\theta + \cos 2\theta$ at the point when $\theta = 0$ in the Cartesian coordinate system is ___(3)___ .

4. Find the **area** of the surface obtained by rotating the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ (parametrized by $x = \cos^3\theta, y = \sin^3\theta$) about the x-axis. ____(4)____



5. Find the **volume** of the solid generated by rotating region enclosed by $y = x^3$ and $y = x$ about the line $y = -1$. ____(5)____