一百學年度第二學期微積分會考試題 (A 卷)

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。試卷加答案卷、答案卡共計 7 頁。

(3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. If $\ln(1-x)=\sum_{n=1}^{\infty}\square$, then $\square=$

(A) $-\dfrac{x^n}{n}$, (B) $(-1)^n\dfrac{x^{2n}}{(2n)!}$, (C) $-\dfrac{x^{2n+1}}{2n+1}$, (D) $(-1)^n\dfrac{x^{2n}}{2^{2n}(n!)^2}$.

2. Which of the following statements is **true**?

(A) If $a_n \ge b_n$ and $\sum a_n$ converges, then $\sum b_n$ converges.

(B) If $a_n \le b_n$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

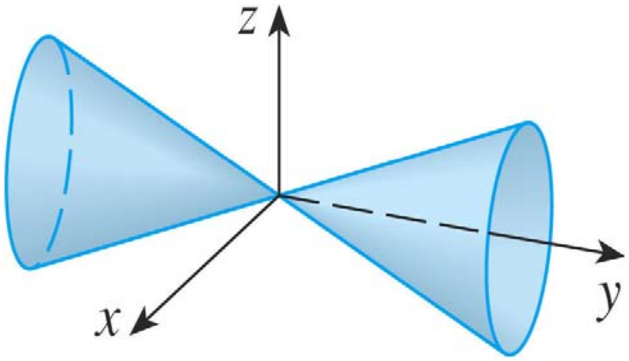
(C) If $|a_n| \ge |b_n|$ and $\sum |a_n|$ converges, then $\sum |b_n|$ converges.

(D) If $|a_n| \le |b_n|$ and $\sum |b_n|$ diverges, then $\sum |a_n|$ diverges.

3. Which one of the following series is **divergent**?

(A) $\sum_{n=1}^{\infty}\dfrac{1}{n\sqrt{n}}$, (B) $\sum_{n=1}^{\infty}\sin\left(\dfrac{\pi}{n}\right)$, (C) $\sum_{n=1}^{\infty}\dfrac{n}{e^n}$, (D) $\sum_{n=1}^{\infty}\left(\dfrac{n}{2n+1}\right)^n$.

4. Find a proper equation for the graph given below



(A) $z = x^2 - y^2$ (B) $x^2 = y^2 + z^2$ (C) $y^2 = x^2 + z^2$ (D) $z^2 = x^2 + y^2$

5. If two vector functions $\boldsymbol{u}(t)$ and $\boldsymbol{v}(t)$ satisfy

$$|\boldsymbol{u}(t) + \boldsymbol{v}(t)| = t^2 + 3$$

and

$$|\boldsymbol{u}(t) - \boldsymbol{v}(t)| = (t-1)^2$$

then $\boldsymbol{u}(t)$ and $\boldsymbol{v}(t)$ are **perpendicular** for $t =$

(A) $2^{1/3}$, (B) -1, (C) $-2^{1/3}$, (D) 0.

6. Which of the following limits **exists**?

(A) $\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$, (B) $\lim_{(x,y)\to(0,0)} \frac{x^2 - \sin^2 y}{x^2 + y^2}$,

(C) $\lim_{(x,y)\to(0,0)} \frac{x^3 + y^2}{x^2 + y^2}$, (D) $\lim_{(x,y)\to(0,0)} \frac{x^3 + y\sin^2 y}{x^2 + y^2}$.

7. Let $f(x, y) = x^4 - 8x^2 + 3y^2 - 6y$. Which of the following statements is **true**?

(A) f has an absolute maximum.
(B) f has two saddle points.
(C) f has two local minima.
(D) f has two local maxima.

8. If $z = y + f(x^2 - y^2)$, where f is differentiable, then $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} =$

(A) 0, (B) x, (C) y, (D) 1.

9. The integral $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-x^2-y^2}\,dydx$ is

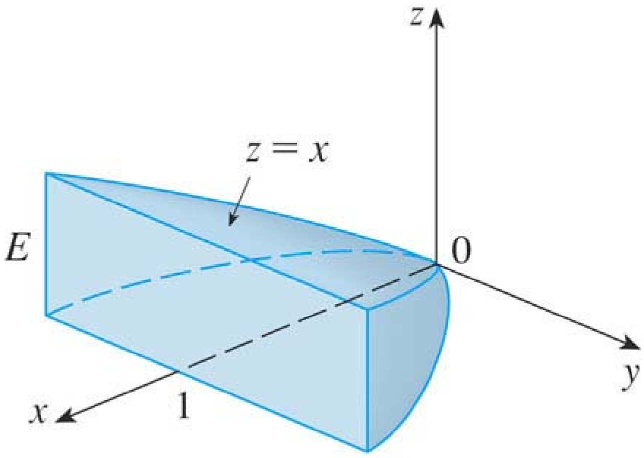
(A) $\frac{\pi}{4}\left(1-e^{-1}\right)$, (B) $\frac{\pi}{3}\left(1-e^{-1}\right)$, (C) $\frac{\pi}{2}\left(1-e^{-1}\right)$, (D) $\pi\left(1-e^{-1}\right)$.

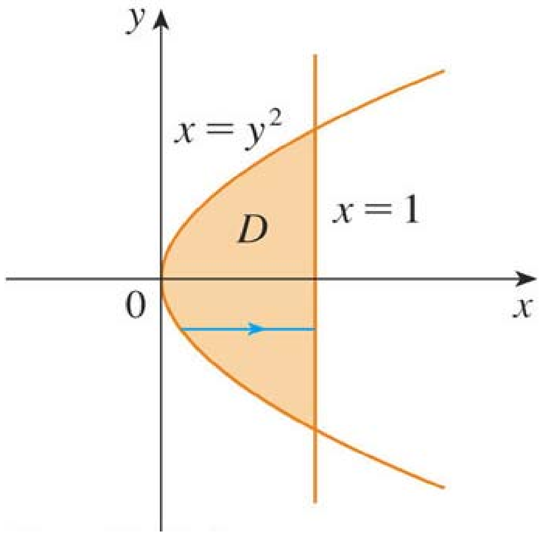
10. Let E be the following solid bounded by $x=y^2$, $x=1$, $z=0$ and $z=x$

If

$$\iiint_E f(x,y,z)\,dV = \int_{-1}^{1}\int_0^1\int_{\square}^1 f(x,y,z)\,dxdzdy,$$

then $\square$ =





(A) 0, (B) y^2, (C) z, (D) none of these.

◎ 多選擇題 （多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣）

11. Which of the following sequences are **convergent**?

(A) $a_n = \left(1+\frac{2}{n}\right)^n$.

(B) $a_n = \ln\left(2n^2+1\right)-\ln\left(n^2+1\right)$.

(C) $\{a_n\} = \{3,10,5,16,8,4,2,1,4,2,1,4,2,1,\ldots\}$.

(D) $a_n = \frac{n!}{2^n}$.

12. Which of the following statements are **always true**?

(A) if $\sum c_n (x-4)^n$ converges at $x=6$, then it converges at $x=1$.

(B) if $\sum c_n (x-4)^n$ diverges at $x=6$, then it diverges at $x=1$.

(C) if $\sum c_n x^n$ converges at $x=6$, then $\sum n c_n x^{n-1}$ converges for $-6 < x < 6$.

(D) if $\sum a_n = A$ and $\sum b_n = B$, then $\sum a_n b_n = AB$.

13. Let S be the quadratic surface given by $x^2 + 2y^2 + 2x - z = 0$. Then, traces of S obtained by intersecting with planes parallel to the coordinate planes are

(A) lines, (B) hyperbolas, (C) ellipses, (D) parabolas.

14. Let

$$f(x,y) = \begin{cases} \dfrac{xy^3}{x^2+y^6}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Which of the following statements are **true**?

(A) f is continuous at $(0,0)$.

(B) $f_x(0,0) = f_y(0,0) = 0$.

(C) $f_x(a,b) = \dfrac{b^3(b^6 - a^2)}{(a^2+b^6)^2}$ for $(a,b) \neq (0,0)$.

(D) f is differentiable at $(0,0)$.

15. The integral $\iiint_E 1dV$, where E is the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\le 1$, is

(A) $\iiint_B abcdV$ where B is the unit ball.

(B) $\frac{\pi abc}{3}$.

(C) $\int_0^1\int_0^{2\pi}\int_0^{\pi} abc\rho^2 \sin\phi d\phi d\theta d\rho$.

(D) $\frac{4\pi abc}{3}$.

填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. The first **three** nonzero terms of the Maclaurin series of $\sqrt{1+x}/(1-x)$ are ____(1)____

2. Let $\boldsymbol{u}(t)=\boldsymbol{i}+t\boldsymbol{j}+2\boldsymbol{k}$ and $\boldsymbol{v}(t)=t\boldsymbol{i}+e^t\boldsymbol{j}+3\boldsymbol{k}$. Then,

$$\int_0^1 u(t)\times v(t)dt = \underline{\qquad (2) \qquad}.$$

3. Let $z=z(x,y)$ be function **implicitly** given by $yz^3+x^2z^2=e^{xyz}$, $z(0,1)=1$.

   Then, $\left.\frac{\partial z}{\partial x}\right|_{(x,y)=(0,1)} = \underline{\qquad (3) \qquad}$.

4. The **maximum** value of $f(x,y,z)=x^2yz+1$ on the intersection of the plane $z=1$ with the sphere $x^2+y^2+z^2=10$ is ____(4)____.

5. Let $D=\left\{(x,y)\middle| x^2+y^2\le 3, y\ge 0\right\}$ be a lamina with density $\rho(x,y)=y$. Then, the center of mass $(\bar{x},\bar{y})=$ ____(5)____.