

一百零一學年度第一學期微積分會考試題 (A 卷)

說明:

- (1) 答題之前請先檢查所取得之試卷與答案卷是否正確。
- (2) 測驗時間 110 分鐘。試卷加答案卷、答案卡共計 7 頁。
- (3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。
- (4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. Consider

$$f(x) = \begin{cases} x^3, & \text{if } x \text{ is rational,} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

Which of the following statements is **NOT TRUE**?

- (A) $f(x)$ is continuous at 0;
(B) $f(x)$ is differentiable at 0;
(C) $f(x)$ is integrable on $[-1,1]$;
(D) $f\left(\frac{\pi}{6}\right) = 0$.

2. Given $f(x) = \frac{1}{2} + \sin(x)$ for $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, what is the value of $(f^{-1})'(0)$?

- (A) 1; (B) $\frac{1}{2}$; (C) -2; (D) $\frac{2}{\sqrt{3}}$.

3. Evaluate $\lim_{n \rightarrow \infty} \frac{1}{n^{16}} [1^{15} + 2^{15} + \cdots + n^{15}] =$

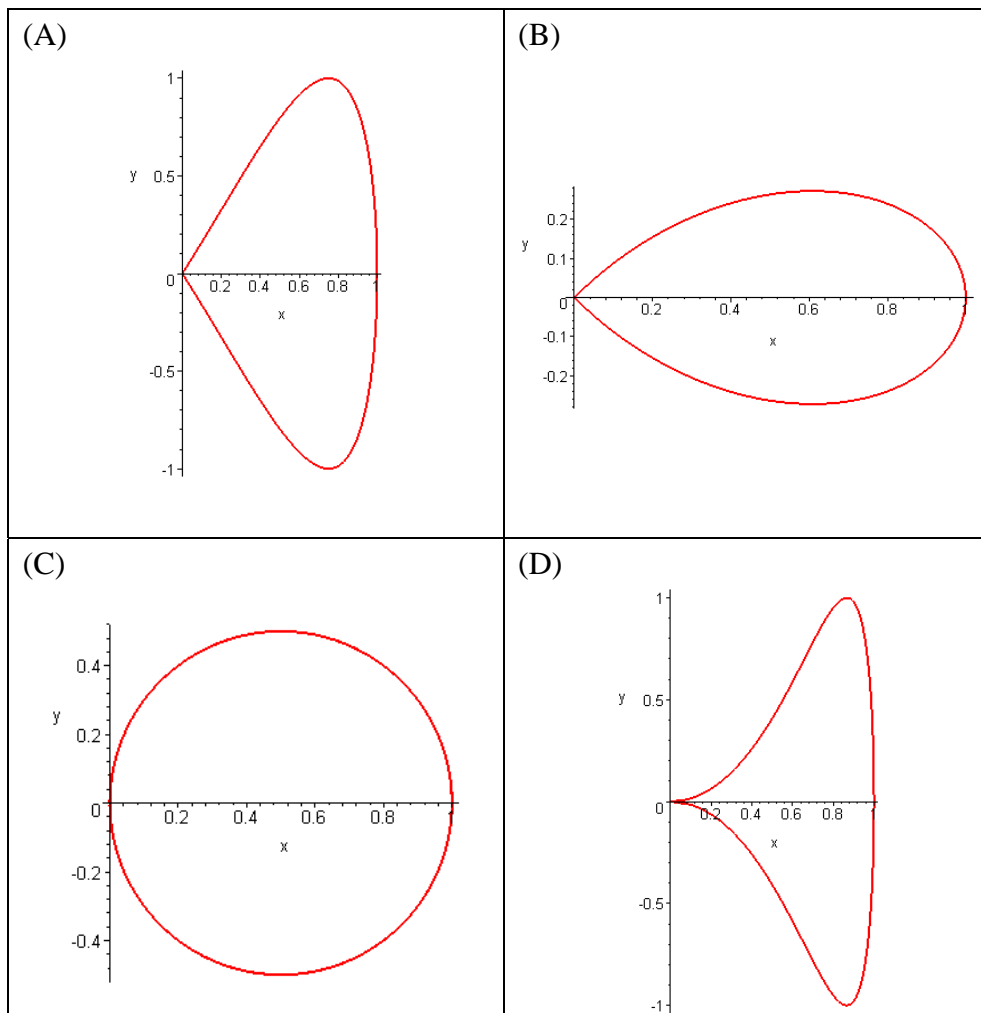
- (A) 0; (B) 1; (C) $\frac{1}{15}$; (D) $\frac{1}{16}$.

4. Evaluate $\lim_{x \rightarrow 0} \frac{\int_0^x \sin t^2 dt}{x^2} =$

- (A) 1; (B) $\frac{1}{2}$; (C) 0; (D) ∞ .

5. Find the graph of the parametric curve whose parametric equation is given by

$$\begin{cases} x = -4t^2 + 4t, \\ y = \sin 2\pi t, \end{cases} \quad 0 \leq t \leq 1.$$



6. Which improper integral is **convergent**?

(A) $\int_0^{\pi} \tan x dx$; (B) $\int_{-\infty}^{\infty} \tan^{-1} x dx$;

(C) $\int_2^{\infty} \frac{1}{x \ln x} dx$; (D) $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$.

7. The **average value** of $f(\theta) = \sec^2\left(\frac{\theta}{2}\right)$ on $\left[0, \frac{\pi}{2}\right]$ is

(A) $\frac{\pi}{4}$; (B) $\frac{4}{\pi}$; (C) 2; (D) $\frac{2}{\pi}$.

8. Find the **area** enclosed by the cardioid $r = 2(1 + \cos \theta)$.

- (A) 2π ; (B) 4π ; (C) 6π ; (D) 8π .

9. Evaluate the definite integral $\int_0^{\frac{\pi}{4}} x \tan^2 x dx =$

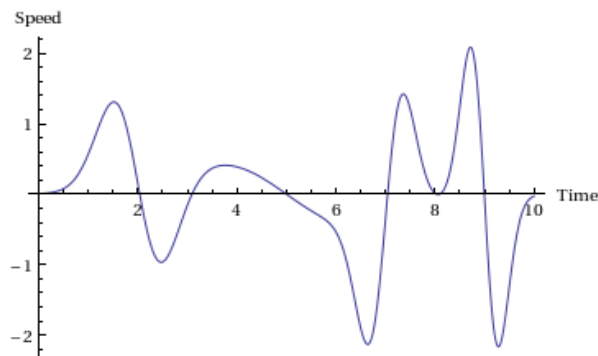
(A) $\frac{\pi}{4} - \frac{\pi^2}{32} - \ln \sqrt{2}$;

(B) $\frac{\pi}{4} - \frac{\pi^2}{32} + \ln \sqrt{2}$;

(C) $\frac{\pi}{4} - \frac{\pi^2}{16} - \ln \sqrt{2}$;

(D) $\frac{\pi}{4} - \frac{\pi^2}{16} + \ln \sqrt{2}$.

10. A particle moves on a line with speed shown in the following graph:



How many turnaround occurs in the particle's motion?

- (A) 4; (B) 5; (C) 6; (D) 7.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

11. Find the equations of the vertical and horizontal asymptotes of the curve

$$y = \frac{x}{x-1} \sin\left(\frac{1}{x}\right).$$

- (A) $x=0$; (B) $x=1$; (C) $y=0$; (D) $y=1$.

12. Let $F(x)$ be an anti-derivative of the function $\frac{1}{2}x^2 + \frac{2}{1+x^2}$. Which of the following points are inflection points of F ?
- (A) -1 ; (B) 0 ; (C) 1 ; (D) $\sqrt{3}$.

13. Which of the following number δ can take such that

$$\text{If } 0 < |x-2| < \delta, \text{ then } |\sqrt{x} - \sqrt{2}| < 1,$$

- (A) $2\sqrt{2} + 1$; (B) $2\sqrt{2} - 1$; (C) 1 ; (D) 2 .

14. Suppose that f and g are continuous functions on \mathbb{R} . Which of the following statements are **true**?

(A) $\int_{-\infty}^{\infty} f(x)dx = \lim_{t \rightarrow \infty} \int_{-t}^t f(x)dx$.

(B) If $\lim_{x \rightarrow \infty} f(x) = 0$, then $\int_1^{\infty} f(x)dx$ is convergent.

(C) If f is an even function and $\int_{-\infty}^{\infty} f(x)dx$ is convergent, then

$$\int_{-\infty}^{\infty} f(x)dx = 2 \int_0^{\infty} f(x)dx.$$

(D) If $0 \leq f(x) \leq g(x)$ for all $x \in \mathbb{R}$ and $\int_{-\infty}^{\infty} f(x)dx$ diverges, then $\int_{-\infty}^{\infty} g(x)dx$ diverges.

15. Which of the following statements are **true**?

(A) The smooth curve $y = f(x)$, $1 \leq x \leq 2$, where $f(x) \leq 3$, is rotated about the horizontal line $y = 3$, then the area of the resulting surface is

$$\int_1^2 2\pi(3 - f(x))\sqrt{1 + (f'(x))^2} dx.$$

(B) The curve $y = x^2$, $0 \leq x \leq 1$, is rotated about the vertical line $x = -2$, then the area of the resulting surface is $\int_0^1 2\pi(2+x)\sqrt{1+4x^2} dx$.

(C) The arclength of the curve $y = \ln(x^2 + 1)$, $1 \leq x \leq e$, is P , and the arclength of the curve $y = 2 \ln x$, $1 \leq x \leq e$, is Q , then $P > Q$.

(D) Suppose $f(x) \geq g(x)$, $a \leq x \leq b$.

Let S = the arclength of the smooth curve $y = f(x)$ on $[a, b]$, and T = the

arclength of the smooth curve $y = g(x)$ on $[a, b]$, then $S \geq T$.

填充題 (五題, 每題五分, 共二十五分, 答錯不倒扣)

1. Two cars start moving from the same point. One travels south at 60 km/h and the other travels west at 80 km/h. **At what rate** is distance between the cars increasing two hours later? _____ (1) _____ km/h

2. The **linear approximation** of

$$g(x) = 3 + \int_1^{x^2} \sec(t-1) dt$$

at $x=1$ is _____ (2) _____.

3. $\int \frac{1}{(x^2+1)^2} dx =$ _____ (3) _____.

4. **The length of the parametric curve** C :

$$\begin{cases} x = \cos t + t \sin t, \\ y = \sin t - t \cos t, \end{cases}$$

from $t=0$ to $t=\pi$ is _____ (4) _____.

5. Let Ω be the region bounded by $y = x^2 - 2x$ and $y = 0$. Then the **volume** of the solid obtained by rotating Ω about y -axis is _____ (5) _____.