一百零一學年度第二學期微積分會考試題 (A 卷)

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。試卷加答案卷、答案卡共計 8 頁。

(3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. Which one of the following series is **conditionally convergent**?

   (A) $\sum_{n=1}^{\infty} \frac{(-2)^n}{3^{n+3}}$;

   (B) $\sum_{n=5}^{\infty} (-1)^n \sin\left(\frac{\pi}{n}\right)$;

   (C) $\sum_{n=1}^{\infty} (-1)^n e^{-\frac{1}{n+1}}$;

   (D) $\sum_{n=4}^{\infty} \sin(n\pi)\sin\left(\frac{\pi}{n}\right)$.

2. Find the **sum** of the series $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\binom{-\frac{1}{2}}{n}}{(2n+1)4^n}$.

   (A) $-\frac{\pi}{3}$; (B) $-\frac{\pi}{6}$; (C) $-\frac{\pi}{12}$; (D) $\frac{\pi}{12}$.

3. If $(a,b,c) \neq (1,0,0)$ is a point on the tangent line to the curve

   $$x = e^t,\ y = te^t,\ z = te^{t^2}$$

   at the point $(1,0,0)$ then the **value** $\frac{a+b-1}{c}$ is

   (A) 0; (B) $\frac{1}{2}$; (C) $\frac{2e-1}{e}$; (D) 2.

4. Determine whether the lines $L_1 : \frac{x+6}{5} = \frac{y}{2} = \frac{z-5}{-1}$, $L_2 : \begin{cases} x = -1 \\ y = 1 - t \\ z = 7 + 3t \end{cases}$ are:

(A) parallel; (B) skew; (C) intersecting; (D) the same.

5. Consider the following function

$$f(x, y) = x^3 + 2xy - y^2 + 1.$$

Which one of the following statements is **TRUE**?

(A) f has 3 critical points.

(B) f has a local minimum at $\left(\frac{-2}{3}, \frac{-2}{3}\right)$.

(C) f has a local minimum at $(0,0)$.

(D) f has a saddle point at $(0,0)$.

6. Consider

$$(1) \lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^4}, \qquad (2) \lim_{(x,y,z)\to(0,0,0)} \frac{x^2 y}{x^2 + y^4 + z^4}.$$

(A) both limits (1) and (2) exist;

(B) first limit (1) exists; second (2) does not;

(C) second limit (2) exists; first (1) does not;

(D) both limits (1) and (2) do not exist.

7. Suppose that $f_x(a,b)$ and $f_y(a,b)$ exist, which one of the followings is **TRUE**.

(A) $\lim_{(x,y)\to(a,b)} f(x, y)$ exists.

(B) f is differentiable at (a,b).

(C) $D_{\vec{u}} f(a,b)$ exists for any unit vector $\vec{u}$.

(D) f may be discontinuous at (a,b).

8. Suppose S is the surface of $y=g(x,z)$ where g is differentiable, and the point $P(x_0,y_0,z_0)\in S$.

Let $G(x,y,z)=g(x,z)-y$ and $S_0=\{(x,y,z)\mid G(x,y,z)=0\}$

Let T be the tangent plane of S at P, and T_0 be the tangent plane of S_0 at P. Which one of the followings is **correct**.

(A) $S\neq S_0$;

(B) The equation for T_0 is

$$g_x(x_0,z_0)(x-x_0)-(y-y_0)+G_z(x_0,y_0,z_0)(z-z_0)=0;$$

(C) $T\neq T_0$;

(D) The equation for T is

$$G_x(x_0,y_0,z_0)(x-x_0)+g_z(x_0,z_0)(y-y_0)-(z-z_0)=0.$$

9. Which is the **value** of the integral $\iint_{\Omega} x^2\,dA$, where $\Omega=\{(x,y)\mid |x|+|y|\leq 1\}$?

(A) $\frac{1}{3}$; (B) $\frac{1}{4}$; (C) $\frac{1}{5}$; (D) $\frac{1}{6}$.

10. Find the **volume** of the region in the first octant bounded by the coordinate planes, the plane $x+y=4$ and the cylinder $y^2+4z^2=16$ is:

(A) $6\pi+\frac{30}{3}$; (B) $8\pi-\frac{32}{3}$; (C) $6\pi-\frac{32}{3}$; (D) $8\pi-\frac{30}{3}$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

11. A sequence $\{a_n\}_{n=0}^{\infty}$ is given by $a_0=2$, $a_{n+1}=5-\frac{4}{a_n}$.

Which of the following statements are **TRUE**?

(A) $\{a_n\}_{n=0}^{\infty}$ is increasing; (B) $\{a_n\}_{n=0}^{\infty}$ is bounded;

(C) $a_2=\frac{10}{3}$; (D) $\lim_{n\to\infty} a_n=4$.

12. Define

$$g(x)=\sum_{n=1}^{\infty}\frac{(x-3)^n}{\sqrt{n}}, \text{ when the series is convergent.}$$

Which of the following statements about $g(x)$ are **TRUE**?

(A) The domain of g is $[2,4]$;

(B) g is differentiable on $(2,4)$;

(C) $g'(3)=\sqrt{2}$;

(D) $g''(3)=\sqrt{2}$.

13. Consider the following function

$$f(x,y)=\begin{cases}\dfrac{(x-y)^2}{x^2+y^2}, & \text{if } (x,y)\neq(0,0);\\ 1, & \text{if } (x,y)=(0,0).\end{cases}$$

Which of the following statements are **TRUE**?

(A) $f_x(0,0)$ and $f_y(0,0)$ both exist.

(B) f is continuous at $(0,0)$.

(C) f is differentiable at $(0,0)$.

(D) f is differentiable at (a,b) with $(a,b)\neq(0,0)$.

14. Let $f:\mathbb{R}^2 \to \mathbb{R}$ be continuous. Which of the following double integrals are the **same**?

(A) $\iint_R f(x,y)\,dA,\ R$ is the region in the first quadrant bounded by the lines $x=2$, $y=2$ and the circle $x^2+y^2=1$;

(B) $\int_0^1\int_{\sqrt{1-x^2}}^2 f(x,y)\,dydx+\int_0^2\int_1^2 f(x,y)\,dxdy$;

(C) $\int_0^2\int_0^2 f(x,y)\,dydx+\int_0^1\int_0^{\sqrt{1-x^2}} f(x,y)\,dydx$;

(D) $\int_0^{\frac{\pi}{4}}\int_1^{\frac{2}{\cos\theta}} f(r\cos\theta,r\sin\theta)\,rdrd\theta+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_1^{\frac{2}{\sin\theta}} f(r\cos\theta,r\sin\theta)\,rdrd\theta$.

15 Which of the following iterated integrals are equal to the iterated integral

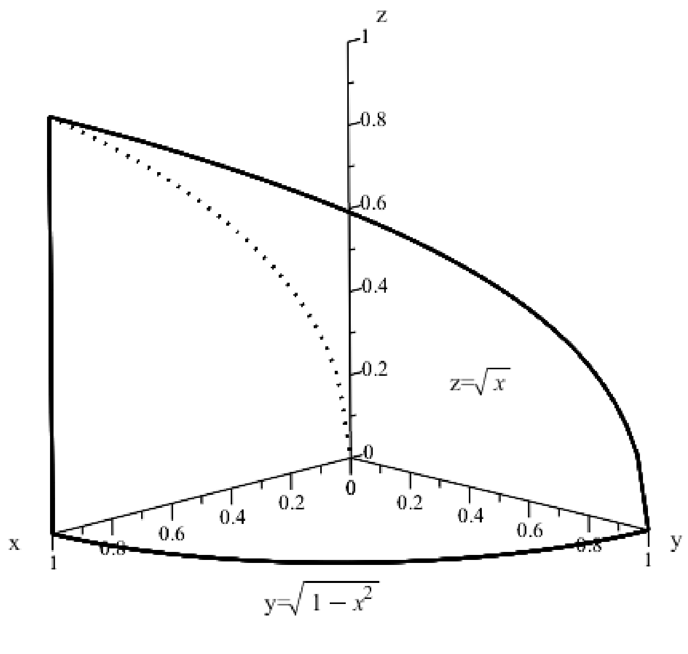
$$\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{x}} f(x,y,z)\,dzdydx;$$

(A) $\int_0^1\int_0^{x^2}\int_0^{1-\sqrt{z}} f(x,y,z)\,dydzdx$;

(B) $\int_0^1\int_0^{\sqrt{x}}\int_0^{\sqrt{1-x^2}} f(x,y,z)\,dydzdx$;

(C) $\int_0^1\int_0^{\sqrt{1-z^4}}\int_{z^2}^{\sqrt{1-y^2}} f(x,y,z)\,dxdydz$;

(D) $\int_0^1\int_0^{\sqrt{1-z^2}}\int_{z^2}^{\sqrt{1-z^2}} f(x,y,z)\,dxdydz$



◎ 填空題 （五題，每題五分，共二十五分，答錯不倒扣）

1. When reparametrize the circle $r(t)=\cos 3t\,\boldsymbol{i}+\sin 3t\,\boldsymbol{j}$ with respect to the **arc length** s measured from $(1,0)$ in the direction of increasing t, we have $r(t(s))=$ ___(1)___.

2. If $z^3x+(x^2+y)z=0$, then $\left.\frac{\partial z}{\partial x}\right|_{(x,y,z)=(0,1,0)}=$ ___(2)___.

3. Find the **minimum positive value** of b such that the series $\sum_{n=1}^{\infty} b^{\ln(n^3+1)}$ is divergent. ___(3)___

4. Evaluate the integral $\iint_R \frac{\sin y}{y}\,dA$, where $R=\{(x,y)\,|\,0\le x\le \pi, x\le y\le \pi\}$.
___(4)___

5. The plane $x+y+2z=2$ intersects the paraboloid $z=x^2+y^2$ in an ellipse. The **point** on this ellipse that is farthest from the origin is ___(5)___.