一百零二學年度第一學期微積分會考試題 (A 卷)

說明:

(1)答題之前請先檢查所取得之試卷與答案卷是否正確。

(2)測驗時間 110 分鐘。試卷加答案卷、答案卡共計 6 頁。

(3)試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4)請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. Find the **maximum** positive number δ such that: If $0<|x|<\delta$ then $\left|e^{-x}-1\right|<\frac{1}{2}$.

   (A) $\ln 3$; (B) $\ln 2$; (C) $\ln\frac{3}{2}$; (D) $\ln\frac{2}{3}$.

2. Consider $f(x)=7x+\cos x$. Which of the following statements is **true** ?
   (A) $f(x)$ has infinitely many roots;
   (B) $f(x)$ is a periodic function;
   (C) $f(x)$ has exactly one root;
   (D) $f(x)$ has a slant asymptote $y=7x$.

3. Evaluate the limit $\lim_{x\to 0}\frac{\int_0^{x^2}\sin\frac{1}{t}\,dt}{x}$.

   (A) 0; (B) 1; (C) 2; (D) ∞.

4. The improper integral $\int_0^{\infty} x^2 e^{-x}\,dx$ =

   (A) -5; (B) 1; (C) 2; (D) $+\infty$.

5. The **area** of the region bounded by the curves $y=e^{2x}$, $y=x$, $x=0$, and $x=1$ is

   (A) $\frac{1}{2}e^2-\frac{1}{2}$; (B) $\frac{1}{2}e^2-1$; (C) $e^2-\frac{3}{2}$; (D) $e-\frac{3}{2}$.

6. Let f be a continuous function on $[1,4]$ and $\int_1^4 f(x)\,dx = 10$.

   Which of the following statements is **always true** ?

   (A) The average value of f on the interval $[1,4]$ is equal to 3;

   (B) The maximum value of f on the interval $[1,4]$ is less than 4;

   (C) The minimum value of f on the interval $[1,4]$ is greater than 0;

   (D) The maximum value of f on the interval $[1,4]$ is greater than 3.

7. Find the **surface area** generated by rotating the parametric curve C about the ***x*-axis**, where C is

$$\begin{cases} x = e^t - t\ , \\ y = 4e^{\frac{t}{2}}\ , \end{cases} \qquad 0 \le t \le 2.$$

   (A) $16\pi(\frac{1}{3}e^3 + e - \frac{4}{3})$; (B) $8\pi(\frac{1}{3}e^3 + e - \frac{4}{3})$;

   (C) $2\pi(e^4 - 9)$; (D) $\pi(e^4 - 9)$.

8. Find the equation of the **tangent** to the curve $x = cos\,t + cos\,2t,\ y = \sin t + \sin 2t,$ at $t = \frac{\pi}{2}$.

   (A) $y = 2x + 3$; (B) $y = 2x - 1$;

   (C) $y = -2x - 1$; (D) $y = -2x + 3$.

9. Find the **area** of the region that lies outside the circle $r = \frac{1}{2}$ and inside the four-leaved rose $r = \cos 2\theta$.

   (A) $\frac{\pi}{4}$; (B) $\frac{\pi}{6} + \frac{\sqrt{3}}{4}$; (C) $\frac{\pi}{8} + \frac{\sqrt{2}}{4}$; (D) $\frac{\pi}{12}$.

10. Let $f(x) = (\ln x)^{sin\,x}$. Then $f'(e) =$

    (A) 0; (B) $\frac{\cos e}{e}$;

    (C) $\cos e + \sin e$; (D) $\frac{\sin e}{e}$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

11. Which of the following statements are **true** ?

(A) $\lim_{x\to\frac{\pi}{2}}\frac{\sin x}{x}=0$; (B) $\lim_{x\to\infty}\frac{\sin x}{x}=1$;

(C) $\lim_{x\to 0}\frac{\sin x}{x}=1$; (D) $\lim_{x\to -\infty}\frac{\sin x}{x}=0$.

12. Which of the following statements are **true** ?

(A) If f is continuous at a, so is $|f|$;

(B) If $x=a$ is a vertical asymptote of $y=f(x)$, then f may or may not be defined at a;

(C) If f is differentiable at a, then f is continuous at a;

(D) If f is defined on $[0,\infty)$ and has no horizontal asymptote, then $\lim_{x\to\infty} f(x)=\infty$ or $\lim_{x\to\infty} f(x)=-\infty$.

13. Suppose that $f'(a)=1$ for some constant a. The **possible values** for the limit $\lim_{x\to a}\frac{f(x)-f(a)}{\sqrt[3]{x}-1}$ are

(A) 0; (B) 1; (C)2; (D) 3.

14. Consider

$$f(x)=\begin{cases}0, & \text{if } x=0,\\ x\ln|x|, & \text{if } x\neq 0.\end{cases}$$

Which of the following statements are **true**?

(A) $f(x)$ is continuous at $x=0$;

(B) $f(x)$ has a local minimum value;

(C) $f(x)$ has a local maximum value;

(D) $f(x)$ has an inflection point.

15. Which of the following statements are **true** ?

(A) If f is continuous on [0,1], then $\int_0^1 f(x)dx = \int_0^1 f(1-x)dx$;

(B) If $2 \le f(x) \le 4$ for $0 \le x \le 1$, then $1 < \int_0^1 f(x)dx < 5$;

(C) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan x}{1+x^2}\, dx = \sqrt{3}\pi$;

(D) $\int_1^e \frac{\ln(x^2)}{x} dx = 1$.

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. The **tangent** line to the curve $\left(x^2+y^2-1\right)^3 = x^3+y-1$ at $(1,0)$ is

   ___(1)___.

2. The **slant** asymptote of $f(x) = \frac{9x^4+x^3+6x^2+5}{x^3+2x^2+x+5}$ is ___(2)___.

3. The integral $\int_0^1 \frac{x-4}{(x+1)\left(x^2+4\right)}\, dx$ = ___(3)___.

4. Let

$$f(x) = \begin{cases} \dfrac{\sin x}{x}, & \text{if } 0 < x \le \pi, \\ 1, & \text{if } x = 0. \end{cases}$$

   The **volume** of the solid obtained by rotating the region bounded by $y = f(x)$, the x-axis, and the y-axis about **the y-axis** is ___(4)___.

5. Set $f(x) = \frac{1}{x[\ln(x+1)]^p}$. Determine **all positive values** of p for which the improper integral $\int_1^\infty f(x)\, dx$ **converges**. ___(5)___.