一百零二學年度第二學期微積分會考試題 (A 卷)

說明:

(1)答題之前請先檢查所取得之試卷與答案卷是否正確。

(2)測驗時間 110 分鐘。試卷加答案卷、答案卡共計 7 頁。

(3)試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4)請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. Let $\gamma(t) = \langle 2t,\ t^2,\ \frac{t^3}{3} \rangle$. Then $\gamma(1) \cdot \left(\gamma'(1) \times \gamma''(1)\right) =$

   (A) $\frac{4}{3}$; (B) 1; (C) $\frac{2}{3}$; (D) $\frac{1}{3}$.

2. The interval of convergence for $\sum_{n=1}^{\infty} \frac{(5x-4)^n}{n^3}$ is

   (A) $(\frac{2}{5},\ \frac{6}{5})$; (B) $\left[\frac{2}{5},\ \frac{6}{5}\right]$; (C) $(\frac{3}{5},\ 1)$; (D) $\left[\frac{3}{5}, 1\right]$.

3. Suppose that a and b are positive integers, such that $f_{xy}(0,0) = -1$, where

$$f(x,y) = \begin{cases} xy\dfrac{x^a - y^b}{x^2 + y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

   Which one of the following statements is **always true** ?

   (A) $a = 2$; (B) $a = 3$; (C) $b = 2$; (D) $b = 3$.

4. Evaluate $\iint_R \sin \pi x \cos \pi y \ dA$, where $R = \left[0, \frac{1}{4}\right] \times \left[\frac{1}{4}, \frac{1}{2}\right]$.

   (A) 1 ; (B) -1 ;

   (C) $\frac{1}{2\pi^2}\left(3 - 2\sqrt{2}\right)$; (D) $\frac{1}{2\pi^2}\left(-3 + 2\sqrt{2}\right)$.

5. Let S be the graph of a differentiable function f; $S=\{(x,y,z)\mid z=f(x,y)\}$.

Suppose that a point $P=(2,1,3)$ lies on the surface S, and the curves γ_1 and γ_2,

$$\gamma_1(t)=\langle 2+3t, 1+8t-t^2, 3-4t+t^2\rangle,$$
$$\gamma_2(u)=\langle 1+u^2, 2u^3-1, 2u+1\rangle,$$

both lie on the surface. Then the value of $f_x(2,1)$ is

(A) 10; (B) -20; (C) 30; (D) -40.

6. Let $f(x,y)=x^4-4xy+8y^2-1$. Which one of the following statements is **true** ?

(A) f has 2 critical points.

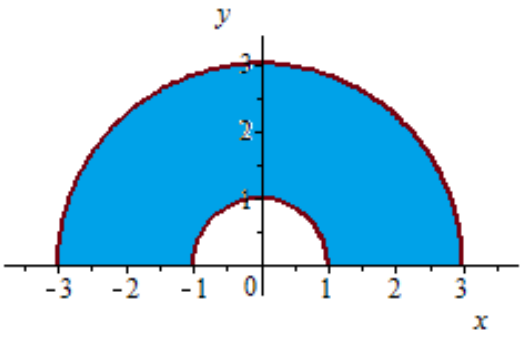
(B) f has a local minimum at (1,1) .

(C) f has an absolute minimum at (1, 1) .

(D) f has a saddle point at (0,0) .

7. Evaluate $\iint_R (3x+4y^2)\,dA$, where R is the region in the upper half-plane bounded by the circles $x^2+y^2=1$, $x^2+y^2=9$ and $y=0$.

(A) 10π ; (B) 20π ; (C) 30π ; (D) 40π .



8. The volume of the solid $\{(x,y,z)|(x+y)^2+(y+z)^2+(z+x)^2\le 1\}$ is

(A) $\frac{\pi}{3}$, (B) $\frac{2\pi}{3}$, (C) π, (D) $\frac{4\pi}{3}$.

9. Which one of the following series is **convergent** ?

(A) $\sum_{n=1}^{\infty} \frac{n^2-3}{n^3+5}$; (B) $\sum_{n=1}^{\infty} ln\left(\frac{n}{3n+1}\right)$;

(C) $\sum_{n=1}^{\infty} \frac{\sin n}{1+(1.5)^n}$; (D) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+2}$.

10. The volume of the solid that lies above the cone $z = \sqrt{3x^2 + 3y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$ is

(A) $\frac{7\pi}{96}$; (B) $\frac{3\pi}{16}$; (C) $\frac{\pi}{32}$; (D) $\frac{\pi}{96}$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

11. Let $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0), \end{cases}$ and $B = \{(x,y) | \, x^2 + y^2 \leq 1\}$.

Which of the following statements are **true** ?

(A) f is continuous; (B) $f_x(0,0)$ exists;

(C) f is differentiable; (D) $\iint_B f(x,y)\, dA = 0$.

12. Let function $f(x,y) = \ln(x^2y^2)$ and point $P = (1,2)$.

Which of the following statements are **true** ?

(A) The gradient of f at P is $\langle 2,1 \rangle$.

(B) The directional derivative of f at P in the direction $\frac{1}{\sqrt{5}}\langle 2,1 \rangle$ is 5.

(C) The maximum rate of change of f at P occurs in the direction $\frac{1}{\sqrt{5}}\langle 2,1 \rangle$.

(D) The maximum rate of change of f at P is 5.

13. Which of the following statements are **true** ?

(A) If $\lim_{n\to\infty} a_n = 0$, then the series $\sum a_n$ is convergent.

(B) The ratio test can be used to determine that the series $\sum_{n=0}^{\infty} \frac{1}{n!}$ converges.

(C) If both of the sequences $\{a_n\}$ and $\{b_n\}$ are divergent, then the sequence $\{a_n b_n\}$ is also divergent.

(D) If $a_n > 0$ and the series $\sum a_n$ converges, then the series $\sum (-1)^n a_n$ also converges.

14. Set $c = \iint_D \left(x^2 + y^2\right) dA$, where $D = \{(x, y) | x^2 + y^2 \leq 1, x \leq y\}$. Which of the following integrals are equal to c ?

(A) $\iint_{D_1} \left(x^2 + y^2\right) dA$, where $D_1 = \{(x, y) | x^2 + y^2 \leq 1, x \geq 0\}$.

(B) $\iint_{D_2} \left(x^2 + y^2\right) dA$, where $D_2 = \{(x, y) | x^2 + y^2 \leq 1, y \geq 0\}$.

(C) $\iint_{D_3} x^2 dA$, where $D_3 = [0,1] \times \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

(D) $\iint_{D_4} x^3 dA$, where $D_4 = [0,1] \times \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

15. Which of the following statements are TRUE ?

(A) $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4} = 0$;

(B) $\lim_{(x,y)\to(0,0)} \frac{3xy^2}{x^2+y^2} = 0$;

(C) $\lim_{(x,y)\to(0,0)} (x^2 + y^2)\ln(x^2 + y^2) = 0$;

(D) $\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = 1$.

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. Let $w(x, y, z)$ satisfy $yw + xw + 3z^2 + zw - 6xz = 0$.

Then $\left.\frac{\partial w}{\partial z}\right|_{(x,y,z)=(1,1,1)} =$ ___(1)___.

2. The **shortest distance** between the points on the surface $S: x^3y^2z = 2$ and the origin is ___(2)____.

3. The **arc length** of $\gamma(t) = \langle e^{3t}\cos 4t, e^{3t}\sin 4t, e^3\rangle$, $0 \le t \le \ln 2$, is ___(3)___

4. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\pi^{2n+1}}{4^{2(n+1)}(2n+1)!}$ = ___(4)____.

5. The **volume** of the solid $\{(x,y,z)||x| + |2y| + |3z| \le 1\}$ is __(5)____.