一百零三學年度第一學期微積分會考試題 (A 卷)

說明:

(1)答題之前請先檢查所取得之試卷與答案卷是否正確。

(2)測驗時間 110 分鐘。試卷加答案卷、答案卡共計 6 頁。

(3)試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4)請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. The **tangent** to the astroid $x(t) = 2\cos^3(t), y(t) = 2\sin^3(t)$ at $t = \pi/4$ is

   (A) $y = -x + \frac{\sqrt{2}}{2}$; (B) $y = -x + \sqrt{2}$; (C) $y = x + \frac{\sqrt{2}}{2}$; (D) $y = x + \sqrt{2}$.

2. How many real **roots** does the equation $4x^5 + x^3 + 2x + 4 = 0$ have?

   (A) 1; (B) 2; (C) 3; (D) 5.

3. Let $f(x) = e^{\tan(x)}$. Then $f'(x)$=?

   (A) $e^{\tan(x)}$; (B) $e^{\tan(x)}\tan(x)$;

   (C) $e^{\tan(x)}\sec(x)\tan(x)$; (D) $e^{\tan(x)}\sec^2(x)$.

4. For what values of a does the curve $y = ax^3 + e^x$ have no **inflection point** ?

   (A) -1; (B) 0; (C) 1; (D) 2.

5. How many horizontal, vertical and slant **asymptotes** does the function

   $f(x) = \frac{\sqrt{3x^2+1}}{3x-5}$ has ?

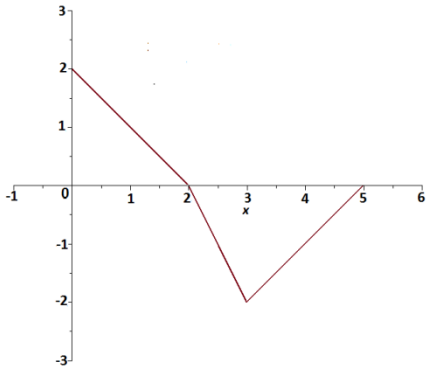
   (A) 1; (B) 2; (C) 3; (D) 4.

6. $(\sin 2x)^{(104)}$=?

   (A) $2^{104}\sin 2x$; (B) $2^{104}\cos 2x$;

   (C) $\sin 2x$; (D) $\cos 2x$.

7. Let f be the function where graph is given below and let $\mathrm{F(x)} = \int_3^x f(t)dt$.



Which of the following values is **largest**?

(A) $\mathrm{F}(0)$; (B) $\mathrm{F}(1)$;

(C) $\mathrm{F}(2)$; (D) $\mathrm{F}(3)$.

8. Let $f(x) = \begin{cases} x|x| & \text{if } -1 \le x \le 1 \\ x^2 & \text{if } x > 1 \\ x^3 & \text{if } x < -1 \end{cases}$. Then f is not **differentiable** at x=?

(A) -1; (B) 0; (C) 1; (D) 2.

9. Let $R = \left\{(x,y) | \left(x - 1 - \sqrt{2}\right)^2 + (y-1)^2 \le 1\right\}$ and S be the solid obtained by rotating R about the line $x = y$. The **volume** of S is

(A) π^2; (B) $2\pi^2$; (C) $3\pi^2$; (D) $4\pi^2$.

10. Find the **area** of the surface generated by revolving the curves $x = \dfrac{e^y + e^{-y}}{2}$, $0 \le y \le \ln 2$, about the y-axis.

(A) $\frac{9}{16}\pi$; (B) $\frac{9}{8}\pi$; (C) $\pi\left(\dfrac{15}{16} + \ln 2\right)$; (D) $\pi\left(\dfrac{15}{8} + 2\ln 2\right)$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

11. Let $f(x) = x^2$. Then f is continuous and so $\lim_{x \to a} f(x) = a^2$ for every $a \in \mathrm{R}$. This means that for every $\epsilon > 0$, there exists a $\delta > 0$ such that

$$|f(x) - a^2| < \epsilon,$$

whenever

(A) $|x - a| < \delta$; (B) $0 < |x - a| < \delta$;

(C) $|x - a^2| < \delta$; (D) $0 < |x - a^2| < \delta$.

12. Let

$$f(x)=\begin{cases} 0 & \text{if } x<0 \\ x & \text{if } 0\le x\le 1 \\ 2-x & \text{if } 1<x\le 2 \\ 0 & \text{if } x>2 \end{cases}$$

and $\mathrm{g}(x)=\int_0^x f(t)dt.$

Which of the following statements are **true** ?

(A) g is continuous on all real numbers;

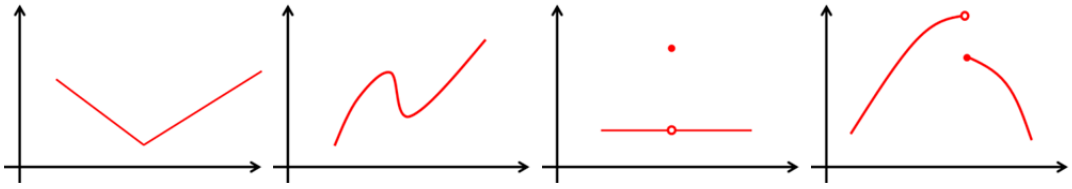
(B) g is differentiable on all real numbers;

(C) f is continuous on all real numbers ;

(D) f is differentiable on all real numbers.

13. Which figures have exactly one **critical** point?

(A) (B) (C) (D)



14. Let $f:[0,1]\to R$ be a one-to-one continuous function satisfying $f(0)=0$, $f(1)=1$ and g be the inverse function of f. Let S be the solid obtained by rotating the region $\{(x,y)|0\le x\le 1,0\le y\le f(x)\}$ about the x-axis. Which of the following integrals denotes the **volume** of S.

(A) $\pi\int_0^1 f^2(x)dx$; (B) $\pi\int_0^1 \mathrm{g}^2(y)dy$;

(C) $2\pi\int_0^1 y\mathrm{g}(y)dy$; (D) $2\pi\int_0^1 y(1-\mathrm{g}(y))dy.$

15. For which values of a does the improper integral $\int_0^\infty \frac{x^a}{1+x^2}dx$ converge?

(A) $a=-1$; (B) $a=-\frac{1}{2}$; (C) $a=\frac{1}{2}$; (D) $a=1.$

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. Find the limit $\lim_{x\to\infty}\frac{x^{2015}(\ln x)^{14}}{e^x} =$ ___(1)___.

2. If $\lim_{x\to\infty}\left(\frac{x+a}{x-a}\right)^x = e$, then $a =$ ___(2)___.

3. $\int_0^{e-2}\ln(2+x)dx =$ ___(3)___.

4. The **arc length** of the curve $y=\frac{1}{3}(\sqrt{x})^3-\sqrt{x}$ with $x\in[0,1]$ is ___(4)___.

5. $\int\frac{1}{x^8-x}dx =$ ___(5)___.