一百零三學年度第二學期微積分會考試題 (A 卷)

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。試卷加答案卷、答案卡共計 7 頁。

(3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題　(單選十題，每題五分，共五十分，答錯不倒扣)

1. If $f(x) = e^{x^3}$, then $f^{(3n)}(0) =$

   (A) 0;　(B) $n!$;　(C) $(3n)!$;　(D) $\frac{(3n)!}{n!}$.

2. The **limit** of the sequence $\left\{\sqrt{3}, \sqrt{3\sqrt{3}}, \sqrt{3\sqrt{3\sqrt{3}}}, \cdots\right\}$ is

   (A) 0;　(B) 1;　(C) 2;　(D)3.

3. Consider

$$f(x,y) = \begin{cases} \dfrac{xy^2}{x^2+y^2}, & \text{if } (x,y) \neq (0,0); \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

   Then the directional derivative $D_u f(0,0)$, where $\mathbf{u} = \langle 1/\sqrt{2}, 1/\sqrt{2} \rangle$, equals

   (A) 0;　(B) $\frac{\sqrt{2}}{4}$;　(C) $\frac{\sqrt{2}}{2}$;　(D) $\sqrt{2}$.

4. The equation of the **tangent plane** to the surface $\cos(xyz) = x^2y^2 + z$ at the point $(1, -1, 0)$ is

   (A) $z = -2x + 2y + 4$;

   (B) $z = -x + y + 2$;

   (C) $z = \frac{1}{2}x - \frac{1}{2}y - 1$;

   (D) $z = \frac{1}{4}x - \frac{1}{4}y - \frac{1}{2}$.

5. Consider the function

$$f(x,y) = -\frac{1}{4}x^4 + \frac{2}{3}x^3 + 4xy - y^2.$$

Which one of the following statements is **true** ?

(A) $f(x,y)$ has exactly four critical points;
(B) $f(x,y)$ has exactly one local minimum;
(C) $f(x,y)$ has exactly two local maximum;
(D) $f(x,y)$ has exactly two saddle points.

6. Let E be a region in $\mathbb{R}^3$ for which the value of the triple integral

$$\iiint_E (1 - x^2 - 2y^2 - 3z^2)dV$$

is a **maximum** . Then E is

(A) $\left\{(x,y,z)\middle| -1 \le x \le 1, -\frac{1}{\sqrt{2}} \le y \le \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{3}} \le z \le \frac{1}{\sqrt{3}}\right\}$;

(B) $\{(x,y,z)| -\sqrt{1-2y^2-3z^2} \le x \le \sqrt{1-2y^2-3z^2}, 2y^2+3z^2 \le 1\}$;

(C) $\{(\rho,\theta,\emptyset)|0 \le \rho \le 1, 0 \le \theta \le 2\pi, 0 \le \emptyset \le \pi\}$;

(D) $\left\{(r,\theta,z)\middle| 0 \le r \le 1, 0 \le \theta \le 2\pi, -\frac{1}{\sqrt{3}} \le z \le \frac{1}{\sqrt{3}}\right\}$.

7. The iterated integral $\int_0^1 \int_y^1 (1+x^2)^{-1} dxdy$ =

(A) $\tan^{-1} 1$; (B) $\ln 2$; (C) $\frac{\tan^{-1} 1}{2}$; (D) $\frac{\ln 2}{2}$.

8. The **area** of the region $\{(x,y)|(x+y)^2 + |x-y| \le 1\}$ is

(A) $\frac{2}{3}$; (B) $\frac{4}{3}$; (C) 2; (D) $\frac{8}{3}$.

9. Let $E = \{(x,y,z) | 1 \le x^2 + y^2 + z^2 \le 4\}$. Then the triple integral

$$\iiint_E \sin[(x^2+y^2+z^2)^{\frac{3}{2}}]\ dV =$$

(A) $4\pi(\cos 1 - \cos 8)$;

(B) $\left(\frac{\pi}{2}\right)^2 (\cos 1 - \cos 4)$;

(C) $\frac{4\pi}{3}(\cos 1 - \cos 8)$;

(D) $\frac{\pi^2}{3}(\cos 1 - \cos 4)$.

10. Let $D = \{(x,y) | x + y \le 1, x \ge 0, y \ge 0\}$. Then the double integral

$$\iint_D e^{x+y}\ \ dA =$$

(A) 1 ; (B) 2 ; (C) $\frac{e}{2}$; (D) e .

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

11. For a particle moving in space $r(t) = (\cos(3t), \sin(3t), 3t)$, which of the followings are **true** ?

(A) $\mathrm{r}(t) \cdot r'(t) = 0$;

(B) $r'(t) \cdot r''(t) = 0$;

(C) The length of the arc between the points $r(a)$ and $r(b)$, where $a < b$, is $3\sqrt{2}(b-a)$;

(D) $\int_0^{\pi} r(t)\, dt = \left(0, -\frac{2}{3}, \frac{3}{2}\pi^2\right)$.

12. Which of the following series are **convergent** ?

(A) $\sum_{n=1}^{\infty} \frac{\sin 4n}{4^n}$; (B) $\sum_{n=1}^{\infty} \frac{1}{n + n\cos^2 n}$;

(C) $\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n}\right)$; (D) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n}$.

13. Which of the following statements are always **true** ?

(A) If $\sum_{n=1}^{\infty}(|a_n|+a_n)$ converges, then $\sum_{n=1}^{\infty}a_n$ converges;

(B) If both $\sum_{n=1}^{\infty}a_n$ and $\sum_{n=1}^{\infty}b_n$ are convergent series with positive terms, then $\sum_{n=1}^{\infty}a_nb_n$ is convergent;

(C) If $a_n>0$ and $\lim_{n\to\infty}n^2a_n=0$, then $\sum_{n=1}^{\infty}a_n$ is convergent;

(D) If $\sum_{n=1}^{\infty}a_n$ is divergent, then $\sum_{n=1}^{\infty}{a_n}^2$ is divergent.

14. Which of the following statements are always **true** ?

(A) If $f(x,y)$ is continuous at (a,b), then $\lim_{(x,y)\to(a,b)}f(x,y)=f(a,b)$.

(B) If $f_x(a,b)$ and $f_y(a,b)$ both exist, then $f(x,y)$ is continuous at (a,b).

(C) There exists a function f such that $f_x(x,y)=x^2+xy^2$, $f_y(x,y)=x^2y+2y^2$;

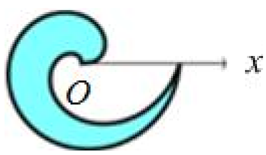
(D) If the partial derivatives f_x and f_y exist, then the directional derivative $D_uf(x,y)=\nabla f(x,y)\cdot u$, where $\nabla f(x,y)$ is the gradient of f at (x,y).

15. Which values of *b* the series $\sum_{n=2}^{\infty}(b^2)^{\ln n}$ is **convergent** ? $(e\approx 2.71828)$

(A) -10 ;　　(B) $-\frac{1}{2e}$;　　(C) 0 ;　　(D) $\frac{1}{3}$.

◎ 填空題　(五題，每題五分，共二十五分，答錯不倒扣)

1. The **area** of the region bounded by $r=\left(\frac{\theta}{2\pi}\right)^2$ and $r=\sqrt{\frac{\theta}{2\pi}}$ with $0\le\theta\le 2\pi$ is ____(1)____



2. Let $x=r\cos\theta$ and $y=r\sin\theta$. Then $\frac{\partial x}{\partial r}\cdot\frac{\partial y}{\partial \theta}-\frac{\partial x}{\partial \theta}\cdot\frac{\partial y}{\partial r}=$ ____(2)____

3. Consider

$$f(x,y)=\sqrt{x^2+y^2}\cos\left(yxe^{(x^2+y^2)^{3/2}}\right).$$

Then $f_x(-1,0)=$ ____(3)____

4. Let R be the square with vertices (0,0), $(1, 1)$, $(1, -1)$ and (2,0). Then

$$\iint_R xydA =$$

_____(4)______

5. The volume of the solid bounded by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$ is ______(5)_____