一百零四學年度第一學期微積分會考試題 (A 卷)

說明:

(1)答題之前請先檢查所取得之試卷與答案卷是否正確。

(2)測驗時間 110 分鐘。試卷加答案卷、答案卡共計 6 頁。

(3)試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4)請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. Let $f(x) = x^3 + ax^2 + bx + c$. If f has local extrema at $x = -2$ and $x = 1$, then which one of the following **must be True**.

   (A) $a + b = \frac{15}{2}$;

   (B) $a + b = -\frac{9}{2}$;

   (C) $a + b + c = -1$;

   (D) $4a - 2b + c = 8$.

2. The **area** enclosed by the curve $x(t) = 1 + e^t,\ y(t) = t - t^2$ and the x-axis is

   (A) $3 + e$; (B)$e - 1$; (C) $3 - e$; (D) $e - 2$.

3. The integral

$$\int \sin(x)\cos(x)dx$$

   is

   (A) $\frac{\cos(2x)}{4} + c$; (B) $-\frac{\sin(2x)}{2} + c$;

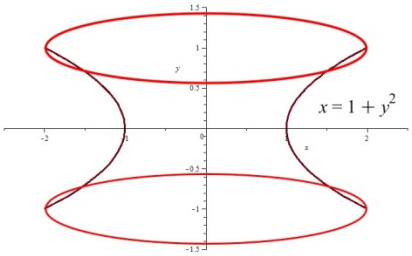
   (C) $\frac{\cos^2(x)}{2} + c$; (D) $\frac{\sin^2(x)}{2} + c$.

4. The **length of** the curve $x(t) = e^t\cos(t), y(t) = e^t\sin(t),\ 0 \le t \le 1,$ is

   (A) $(e - 1)$; (B) $\sqrt{2}(e - 1)$; (C) e^{π}; (D) $\sqrt{2}e^{\pi}$.

5. The **volume** of the solid obtained from rotating the region bounded by the curve $x = 1 + y^2,\ -1 \le y \le 1$, and the *y*-axis about the *y*-axis is

(A) $\frac{56\pi}{15}$; (B) $\frac{28\pi}{15}$; (C) $\frac{56\pi}{5}$; (D) $\frac{28\pi}{5}$.



6. The limit $\lim_{x\to 0} \frac{x \sin x}{1-\cos(2x)} =$

(A) 2; (B) 1; (C) $\frac{1}{2}$; (D) 0.

7. Let $F(x) = \int_{x^3}^{1} \sqrt{2^t + 14}\ dt$. Then the derivative $(F^{-1})'(0) =$

(A) $-\frac{1}{12}$; (B) $-\frac{1}{2}$; (C) 0; (D) $\frac{1}{4}$.

8. The **surface area** of the figure obtained by rotating $y = x^3,\ 0 \le x \le 1$ about the *x*-axis is

(A) $\frac{\pi}{27}\left((\sqrt{10})^3 - 1\right)$; (B) $\frac{\pi}{12}\left((\sqrt{10})^3 - 1\right)$;

(C) $\frac{\pi}{27}\left((\sqrt{2})^3 - 1\right)$; (D) $\frac{\pi}{12}\left((\sqrt{2})^3 - 1\right)$.

9. Let $g(x) = xe^x$**.** Then the **absolute maximum** value of $g(\sin x + 2\cos x)$, $x \in$ R, is

(A) $\sqrt{3}e^{\sqrt{3}}$; (B) $\sqrt{5}e^{\sqrt{5}}$; (C) $-\frac{1}{e}$; (D) $-\frac{3}{e}$.

10. $\lim_{n\to\infty}\{\frac{1}{n^3\sqrt{4n^2+1}}+\frac{8}{n^3\sqrt{4n^2+4}}+\cdots+\frac{n^3}{n^3\sqrt{4n^2+n^2}}\}=$

(A) $3\sqrt{5}$; (B) $\frac{3}{2}\sqrt{5}$; (C) $-\frac{14}{3}\sqrt{5}+\frac{32}{3}$; (D) $-\frac{7}{3}\sqrt{5}+\frac{16}{3}$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

11. The improper integral

$$\int_0^\infty \frac{\sqrt{x}}{1+x^a}dx$$

converges for

(A) $a=\frac{1}{2}$; (B) $a=\frac{3}{2}$; (C) $a=\frac{5}{2}$; (D) $a=\frac{7}{2}$.

12. Which of the following lines is an (horizontal, vertical, slant) asymptote of $y=\frac{\sqrt{e^{2x}+x^2}}{e^x-1}$.

(A) $x=0$; (B) $y=1$; (C) $y=-1$; (D) $y=x$.

13. Which of the following statements are **True** ?

(A) $\lim_{x\to0}\frac{\tan x-x}{x}=0$; (B) $\lim_{x\to0}x^3\sin\frac{1}{x}=0$;

(C) $\lim_{x\to0^+}(1+x)^{1/x}=1$; (D) $\lim_{x\to1}\frac{\sin(x-1)}{x^2+x-2}=\frac{1}{3}$.

14. Given function $f(x)=\frac{2x+5}{x-7}$, which of the following statements are **True**.

(A) Its domain is $(-\infty,7)\cup(7,\infty)$;
(B) It is an odd function;
(C) Its inverse function $f^{-1}(x)=\frac{7x+5}{x-2}$;
(D) It is a continuous function on its domain.

15. Which of the following statements are **True** ?

(A) $\int_{-\infty}^{\infty} x^3 dx = 0$;

(B) $\int_3^{\infty} \frac{1}{x^{\csc 1}} dx$ is convergent;

(C) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan(x^3)}{1+x^2}\, dx = \sqrt{2}\pi$;

(D) $\int_0^{\frac{\pi}{4}} \tan^2 x\, dx = 1 - \frac{\pi}{4}$.

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. The **slope** of the tangent to $x^3 + xy - \cos(xy) = 0$ at the point $(1,0)$ is ____(1)____.
2. If f is continuous and $\int_0^1 f(x)dx = 4$, then $\int_0^1 (1-x)f(x^2-2x+1)dx =$ ____(2)____.
3. The **derivative** of $\int_x^{x^3} \sin(t^2)\, dt$ is ____(3)____.
4. The equation of the **tangent** to the curve $\begin{cases} x(t) = e^{\sqrt{t}} \\ y(t) = t - \ln(t^2) \end{cases}$ through the point $\big(x(1), y(1)\big)$ is ____(4)____.
5. Let $f'(x) \geq 3$ for all $x \in [1,4]$. Suppose $f(1) = 1$ and $f(4) = 10$. Then $f(2) =$ ____(5)____.