一百零四學年度第二學期微積分會考試題 (A 卷)

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。試卷加答案卷、答案卡共計 6 頁。

(3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. The **slope** of the tangent line of the curve $r = 2 + \sin\theta$ at $\theta = \frac{\pi}{4}$ is

   (A) -1; (B) $-1 - \frac{\sqrt{2}}{2}$; (C) 2; (D) $-2 + \sqrt{2}$.

2. The **area** of the region bounded by $|x|^{\frac{1}{3}} + |y|^{\frac{1}{3}} = 1$ is

   (A) $\frac{1}{20}$; (B) $\frac{1}{10}$; (C) $\frac{1}{5}$; (D) $\frac{1}{2}$.

3. The interval of convergence of $\sum_{n=1}^{\infty}\left[\ln\left(1 + \frac{2}{\sqrt{n}}\right)\right]x^n$ is

   (A) $(-1,1)$; (B) $[-1,1)$; (C) $(-2,2)$; (D) $[-2,2]$.

4. Let $h(t) = \frac{\sin(t)}{t}$, if $t \neq 0$; $h(0) = 1$, and $f(x) = \int_0^x h(t)dt$. Then $f^{(5)}(0)$ =

   (A) $\frac{1}{5}$; (B) $-\frac{1}{4}$; (C) $\frac{1}{3}$; (D) $-\frac{1}{2}$.

5. If $f\colon \mathbb{R} \to \mathbb{R}$ is differentiable, and if $F(x,y) = y + f(x^2 - y^2)$, then

$$y\frac{\partial F(x,y)}{\partial x} + x\frac{\partial F(x,y)}{\partial y} =$$

   (A) 0; (B) x; (C) y; (D) $2xyf'(x^2 - y^2)$.

6. The limit $\lim_{(x,y)\to(0,0)} \frac{x\sin(y)}{x^2+y^2} =$

(A) 0; (B) $\frac{1}{2}$; (C) 1; (D) Does not exist.

7. The iterated integral $\int_0^1 \int_x^1 2\sin(y^2)dydx =$

(A) $\cos 1 - 1$; (B) $1 + \sin 1 - \cos 1$;
(C) $1 - \cos 1$; (D) $1 + \sin 1$.

8. The **surface area** over the disk $x^2 + y^2 \leq 2$ of the circular paraboloid $z = x^2 + y^2$ is

(A) $\frac{52}{27}\pi$; (B) $\frac{5}{6}\pi$; (C) $\frac{13}{3}\pi$; (D) $\frac{\pi}{6}\left(17\sqrt{17} - 1\right)$.

9. Let $a_1 = 0, a_2 = 1$ and $a_{n+1} = \frac{2}{3}a_n + \frac{1}{3}a_{n-1}$ for $n \geq 2$. The **limit** of a_n is

(A) $\frac{1}{3}$; (B) $\frac{1}{2}$; (C) $\frac{2}{3}$; (D) $\frac{3}{4}$.

10. The value of the double integral $\iint_D \frac{x-y}{x+y}\, dA$, where D is the square with verties $(0,2), (1,1), (2,2)$ and $(1,3)$, is

(A) $-\ln(2)$; (B) $-2\ln(2)$; (C) $3\ln(2)$; (D) $6\ln(2)$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

11. Let $f(x, y) = -x^3 + 4xy - y^2 + 1$. Which of the following statements are **true** ?

(A) f has a local minimum at $(\frac{8}{3}, \frac{16}{3})$;

(B) f has a local maximum at (0,0) ;

(C) f has two critical points;

(D) f has one saddle point.

12. Suppose $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ is convergent. Which of the following statements must be **true** ?

(A) $\sum_{n=1}^{\infty} \sqrt{b_n}$ is convergent;

(B) $\sum_{n=1}^{\infty} \sqrt{a_n b_n}$ is convergent;

(C) $\sum_{n=1}^{\infty} a_n b_n$ is convergent;

(D) $\sum_{n=1}^{\infty} (-1)^n a_n b_n$ is convergent.

13. Which of the following series are **divergent**?

(A) $\sum_{n=1}^{\infty} \frac{5}{4n - 3\ln n - 7}$;　　(B) $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$;

(C) $\sum_{n=1}^{\infty} \left(\sqrt[n]{2} - 1\right)^n$;　　(D) $\sum_{n=1}^{\infty} \frac{e^n + n}{e^{2n} - n^2}$.

14. Let $f(x,y) = \begin{cases} \frac{x^2 y}{x^4 + y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$ Which of the following statements are **true** ?

(A) $f_x(0,0) = f_y(0,0) = 0$;

(B) f is continuous at $(0,0)$;

(C) f is differentiable at $(0,0)$;

(D) $D_{\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle} f(0,0) = \frac{3}{2}$.

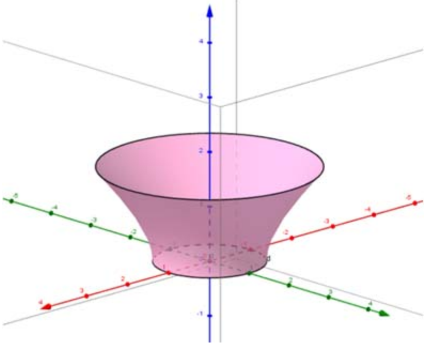
15. Which of the following iterated integrals are equal to the triple integral $\iiint_E f(x,y,z)\,dV$? Where E is the solid bounded by the hyperboloid of one sheet $x^2 + y^2 - z^2 = 1$ and planes $z = \sqrt{3}$ and $z = 0$.

(A) $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2-1}}^{\sqrt{3}} f(x,y,z)\, dzdydx$;

(B) $\int_{0}^{\sqrt{3}} \int_{-\sqrt{z^2+1}}^{\sqrt{z^2+1}} \int_{-\sqrt{z^2-x^2+1}}^{\sqrt{z^2-x^2+1}} f(x,y,z)\, dydxdz$;

(C) $\int_{0}^{\sqrt{3}} \int_{-\sqrt{z^2+1}}^{\sqrt{z^2+1}} \int_{-\sqrt{z^2-y^2+1}}^{\sqrt{z^2-y^2+1}} f(x,y,z)\, dxdydz$;

(D) $\int_{0}^{2\pi} \int_{0}^{1} \int_{\sqrt{r^2-1}}^{\sqrt{3}} f(r\cos\theta, r\sin\theta, z) r\, dzdrd\theta$.



◎ 填空題　(五題，每題五分，共二十五分，答錯不倒扣)

1. The **length** of the curve $r = \cos^3\left(\frac{\theta}{3}\right)$, $0 \le \theta \le \frac{\pi}{4}$, is ____(1)____.

2. The **absolute maximum** of $f(x,y) = -\frac{2y}{x^2+y^2+4}$ on $D = \{(x,y) : x^2+y^2 \le 1\}$ is ____(2)____.

3. Let $f(x,y) = \begin{cases} x^2\tan^{-1}\frac{y}{x} - y^2\tan^{-1}\frac{x}{y} & \text{if } xy \neq 0, \\ 0 & \text{if } xy = 0. \end{cases}$ Then $f_{xy}(0,0) =$ ____(3)____.

4. The **tangent plane equation** of the surface $\sin(xyz) = x + 2y + 3z$ through the point $(2,-1,0)$ is ____(4)____.

5. The **volume** of the solid that lies within the sphere $x^2+y^2+z^2 = 4$, above the xy-plane, and below the cone $z = \sqrt{x^2+y^2}$ is ____(5)____.