

一百零五學年度第一學期微積分會考試題 (A 卷)

說明:

- (1) 答題之前請先檢查所取得之試卷與答案卷是否正確。
- (2) 測驗時間 110 分鐘。試卷加答案卷、答案卡共計 7 頁。
- (3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。
- (4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. How many horizontal, vertical and slant **asymptotes** does the function

$$f(x) = \frac{x^3-1}{x(x+1)} \text{ have?}$$

- (A) 4; (B) 3; (C) 2; (D) 1.

2. Given that $g(3) = 3, g'(3) = 7, h(6) = 3$ and $h'(6) = -2$, and let

$$f(x) = \frac{g(h(x))}{h(x)}. \text{ Then } f'(6) =$$

- (A) $-\frac{8}{3}$; (B) $-\frac{16}{3}$; (C) -2 ; (D) -4 .

3. Find the **derivative** of $f(x) = \ln|x^3 - 4x + 1|$ when $x^3 - 4x + 1 \neq 0$.

(A) $f'(x) = \frac{3x^2-4}{x^3-4x+1}$;

(B) $f'(x) = \frac{3x^2-4}{|x^3-4x+1|}$;

(C) $f'(x) = -\frac{3x^2-4}{|x^3-4x+1|}$;

(D) $f'(x) = -\frac{3x^2-4}{x^3-4x+1}$.

4. The limit $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x} - \frac{2}{x^2}\right)^x =$

- (A) 1; (B) e^{-1} ; (C) e^{-2} ; (D) e^{-3} .

5. How **many points of inflection** does the function $f(x) = x^6 - 15x^2 + 1$ have?
 (A) 0; (B) 1; (C) 2; (D) 4.

6. $\int_0^{\frac{1}{2}\ln 3} e^x \sqrt{1 + e^{2x}} dx =$

- (A) $\frac{1}{2} \left(2\sqrt{3} - \sqrt{2} + \ln \frac{2+\sqrt{3}}{1+\sqrt{2}} \right)$; (B) $2\sqrt{3} - \sqrt{2} + \ln \frac{2+\sqrt{3}}{1+\sqrt{2}}$;
 (C) $\frac{1}{2} \left(2\sqrt{2} - \sqrt{3} + \ln \frac{2+\sqrt{2}}{1+\sqrt{3}} \right)$; (D) $2\sqrt{2} - \sqrt{3} + \ln \frac{2+\sqrt{2}}{1+\sqrt{3}}$;

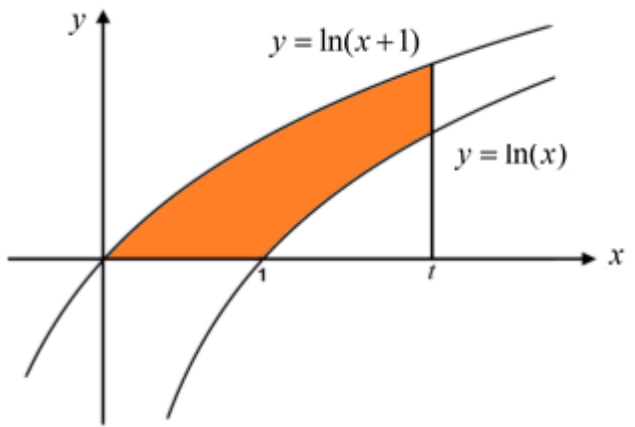
7. If the infinite region $\Omega = \left\{ (x, y) \mid x \geq 1, 0 \leq y \leq \frac{1}{x} \right\}$ is rotated about the **x-axis**,
 how about the **volume** of the resulting solid and its **surface area**?
 (A) Volume is finite. Surface area is finite.
 (B) Volume is infinite. Surface area is finite.
 (C) Volume is finite. Surface area is infinite.
 (D) Volume is infinite. Surface area is infinite.

8. Let R be the region bounded by $y = \cos x$ and the line $y = 1$ with $x \in [0, 2\pi]$.
 The **volume** of the solid obtained by rotating the region R about the line $y = 1$
 is
 (A) π^2 ; (B) $2\pi^2$; (C) $3\pi^2$; (D) $4\pi^2$.

9. The **length** of the curve $x = 3 \cos t - \cos 3t, y = 3 \sin t - \sin 3t, 0 \leq t \leq \pi$ is
 (A) 10; (B) 11; (C) 12; (D) 13.

10. Let R be the region enclosed by $y = \ln x, y = \ln(x + 1), y = 0$, and
 $x = t$ ($t > 1$). If $V(t)$ is the volume of the solid obtained by rotating R about
 the **y-axis**, then the limit $\lim_{t \rightarrow \infty} \left(\frac{d}{dt} V(t) \right) =$

- (A) π ;
 (B) 2π ;
 (C) 3π ;
 (D) 4π ;



◎ 多選擇題 (多選五題, 每題五分, 共二十五分。答錯一個選項扣兩分, 錯兩個選項以上不給分, 分數不倒扣)

11. If $f(x) = \frac{ax}{x^2+b^2}$ has a local minimum at $x = -2$ and $f'(0) = 1$, then a and

b could be

- (A) $a = 4, b = 2$;
- (B) $a = 4, b = -2$;
- (C) $a = 2, b = 4$;
- (D) $a = -2, b = 4$;

12. Which of the following statements are **True** for $f(x) = x^{\frac{2}{3}}(9-x)^{\frac{1}{3}}$?

- (A) f is increasing on $(0, 4)$.
- (B) f has a local minimum at $x = 9$.
- (C) f has a local minimum at $x = 0$.
- (D) f has a local maximum at $x = 6$.

13. Consider $x \in [0,1]$ and $f(x) = \frac{\sin x}{x}$ if $x \neq 0$, $f(x) = 0$ if $x = 0$. Which of the

following statements are **True**?

- (A) $f(x)$ is continuous.
- (B) $f(x)$ is differentiable on $(0,1)$.
- (C) $f(x)$ is integrable.
- (D) $(f \circ f)(x)$ is integrable.

14. Let R be the region bounded **below** by the graph of $y = x^3 - x$ and bounded **above** by the graph of $y = \sin(\pi x)$. Which of the following statements are **True**?

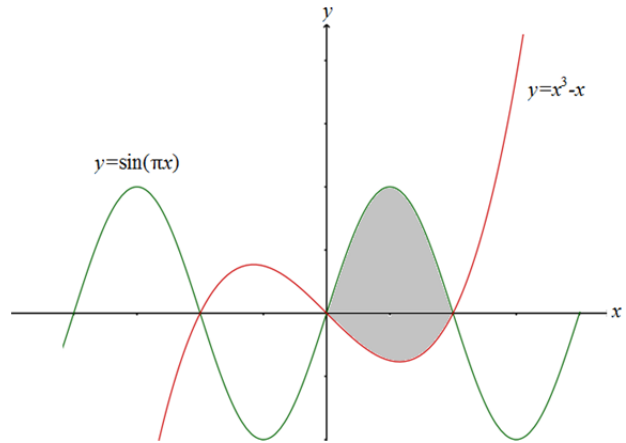
(A) $(0,0)$ and $(\pi,0)$ are on the boundary of the region R .

(B) The area of $R = \int_0^1 (\sin(\pi x) - x^3 + x) dx$.

(C) Let S be a solid with the base R and each cross-section perpendicular to the base R is an equilateral triangle. Then the volume of this solid is equal to

$$\frac{\sqrt{3}}{4} \int_0^1 (x^3 - x - \sin(\pi x))^2 dx.$$

(D) The volume of the solid obtained by rotating the region R about the line $x = -1$ can be evaluated as $2\pi \int_0^1 (x+1)(\sin(\pi x) - x^3 + x) dx$.



15. Let f be the function given by $f(x) = \int_1^x (t^2 - 4t + 3)e^{-t} dt$.

Which of the following statements about f must be **True**?

(A) f is increasing on the interval $(1, 3)$.

(B) f is increasing on the interval $(3, 4)$.

(C) $f(3) > 0$.

(D) $f(1) = 0$.

◎ 填空题 (五题, 每题五分, 共二十五分, 答错不倒扣)

1. The limit $\lim_{x \rightarrow \infty} \frac{x^{2017}}{2^x} = \underline{\quad (1) \quad}$.

2. The **tangent line** to the curve $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at the point $(0, \frac{1}{2})$ is $\underline{\quad (2) \quad}$.

3. The **absolute maximum value** of the function $f(x) = x\sqrt{9-x^2}$, $-3 \leq x \leq 3$, is (3) .

4. Let $f(x) = \int_0^x e^{t^2} dt$. Then $f''(x) =$ (4) .

5. Determine **all values** of p such that $\int_1^\infty x^p |\sin x| dx$ **converges**. (5)