一百零六學年度第一學期微積分會考試題 (A 卷)

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。試卷加答案卷、答案卡共計 6 頁。

(3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ **單選擇題　(單選十題，每題五分，共五十分，答錯不倒扣)**

1. The **slope** of the tangent line to the curve $x^3 + y^3 - 9xy = 0$ at $(2,4)$ is

   (A) $\frac{5}{4}$; (B) $\frac{4}{5}$; (C) $\frac{3}{2}$; (D) $\frac{2}{3}$.

2. Consider the curve defined by the parametric equations, $x = \theta - \sin\theta$ and $y = 1 - \cos\theta$ with $\theta \in [0,2\pi]$. Find the **volume** of the solid obtained by rotating the region bounded by this curve and the x-axis about the x-axis.

   (A) $3\pi^2$; (B) $4\pi^2$; (C) $5\pi^2$; (D) $6\pi^2$.

3. The **length** of the curve $y = \int_0^x \sqrt{\cos 2t}\, dt$ from $x = 0$ to $x = \frac{\pi}{4}$ equals

   (A) $\frac{1}{2}$; (B) 1; (C) $\frac{3}{2}$; (D) 2.

4. Consider the parametric curve, $x = \cos 2t$ and $y = \sin 4t$ for $0 \le t \le 2\pi$. How many points on this curve at which **tangent** lines are **horizontal**?

   (A) 2; (B) 4; (C) 6; (D) 8.

5. Let $f(x) = \begin{cases} ax^3 & \text{if } x \le 1, \\ x^2 + b & \text{if } x > 1. \end{cases}$ If f is differentiable on $\mathbb{R}$, then the ordered pair (a, b) is

   (A) $\left(\frac{2}{3}, \frac{-1}{3}\right)$; (B) $\left(\frac{-1}{3}, \frac{2}{3}\right)$; (C) $\left(\frac{2}{3}, \frac{1}{3}\right)$; (D) $\left(\frac{1}{3}, \frac{2}{3}\right)$.

6. The limit $\lim_{x\to 0+} x^{-\frac{3}{2}} \int_0^{\sqrt{x}} \sin(t^2)\cos(t)dt$ equals

(A) ∞; (B) $\frac{1}{2}$; (C) $\frac{1}{3}$; (D) 0.

7. Let $\alpha = \int_0^1 2^{-x} \sin \pi x \, dx$. Then, the **improper** integral $\int_0^\infty 2^{-x} \sin \pi x \, dx$ equals

(A) $\frac{\alpha}{3}$; (B) $\frac{\alpha}{2}$; (C) $\frac{2\alpha}{3}$; (D) α.

8. The limit $\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$ is

(A) 1; (B) $\frac{1}{2}$; (C) 0; (D) ∞.

9. The limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n+\sqrt{(i-1)i}}$ equals

(A) 2; (B) $\ln 2$; (C) e^2; (D) $\tan^{-1}2$.

10. How many **roots** does the equation $5x + 3\sin x + 9 = 0$ have?

(A) 0; (B) 1; (C) 2; (D) 3;

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

11. Let f be a differentiable **odd** function on $\mathbb{R}$ satisfying $\int_0^1 f(x)dx = 0$. Then,

$\int_0^1 xf'(1-x)dx$ **MUST** equal

(A) $f(1) - f(0)$; (B) $f(1)$; (C) $-f(0)$; (D) 0.

12. Consider the function $f(x) = \int_0^{x^2} \frac{dt}{1+t^4}$. Which of the following statements are **TRUE**?

(A) $f(x)$ is continuous on $\mathbb{R}$.

(B) $f(x)$ has neither local maxima nor local minima.

(C) $f(x)$ is concave upward on some interval and concave downward on some interval.

(D) $f(x)$ has exactly one inflection point.

13. Suppose that $\lim_{x\to 0} f(x) = L$ and $\lim_{x\to 0} g(x) = M$. Which of the following statements are **TRUE**?

(A) If $f(x) < g(x)$ for all $x \neq 0$, then $L < M$.

(B) $\lim_{x\to 0} |f(x)| = |L|$.

(C) If $L = 0$, then $\lim_{x\to 0} g\big(f(x)\big) = M$.

(D) If $f(x) = g(x)$ for all $x \neq 0$, then $L = M$.

14. Which of the following represents the **surface area** of revolution obtained by rotating the curve, $y = \sin x$ with $x \in [0, \pi]$, about the x-axis?

(A) $2\pi \int_0^{\pi} \sin x \sqrt{1 + \cos^2 x}\, dx$.

(B) $2\pi \int_0^{\pi} \cos x \sqrt{1 + \sin^2 x}\, dx$.

(C) $2\pi(\sqrt{2} + \ln(\sqrt{2} + 1))$.

(D) $2\pi(\sqrt{2} + 2\ln(\sqrt{2} + 1))$.

15. Let f be a continuous function on $\mathbb{R}$. Which of the following are **TRUE**?

(A) If $f'(x) > 0$ for all x, then $\lim_{x\to\infty} f(x) = \infty$.

(B) If $f'(x) > 0$ and $f''(x) > 0$ for all x, then $\lim_{x\to\infty} f(x) = \infty$.

(C) If $f'(x) > 0$ and $f''(x) < 0$ for all x, then $\lim_{x\to\infty} f(x) = \infty$.

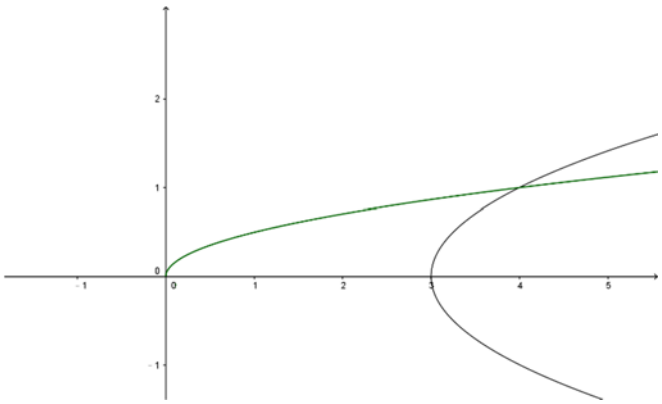
(D) If $f'(x) > 0$ and $f''(x) < 0$ for all x, then there is x such that $f(x) < -20180110$.

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. Let $f(x) = \sqrt{x} \int_0^{\sqrt{x}} e^{xt^2} dt$. Then, $f'(x)$ is ____(1)____.

2. The **slant** asymptote of $f(x) = x + x \sin\frac{1}{x}$ is ____(2)____.

3. The **area** of the region in the first quadrant enclosed by the three curves, $y = 0$, $y = \frac{\sqrt{x}}{2}$ and $x = y^2 + 3$, is ____(3)____.



4. Let $f(x) = \frac{2x}{(x^2+3)^2}$ for $x \in \mathbb{R}$. Then, the **absolute minimum** of f is ____(4)____.

5. If the integral $\int_0^\infty \left(\frac{x}{x^2+1} - \frac{a}{2x+1}\right) dx$ is **convergent**, then $a =$ ____(5)____.