一百零七學年度第一學期微積分會考試題 (A 卷)

說明：

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。試卷加答案卷、答案卡共計 6 頁。

(3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做爲微積分獎給獎依據。

(4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣。)

1. For $x > 0$, let $f(x)$ be the **average** value of e^{-t} on $[0, x]$. How many **critical numbers** on $(0, \infty)$ does f have?

   (A) 0; (B) 1; (C) 2; (D) 3.

2. Let $f(x) = (1 + \frac{a}{x})^{1/x}$, where $a \neq 0$. Then $f'(a) =$

   (A) $\frac{2^{1/a}}{2a}$; (B) $-\frac{2^{1/a}}{2a^2}$; (C) $-\frac{2^{1/a}}{a^2}(\ln 2 - \frac{1}{2})$; (D) $-\frac{2^{1/a}}{a^2}(\ln 2 + \frac{1}{2})$.

3. Let $f(x) = \int_{2x}^{3-x} e^{t^2}\, dt$. Then $(f^{-1})'(0) =$

   (A) $\frac{-1}{3e^4}$; (B) $\frac{1}{e^9 - 1}$; (C) $\frac{-1}{e^9 + 2}$; (D) $\frac{1}{e^4}$.

4. The **value** of $\int_1^{\sqrt{3}} \frac{x^4 - x^3 + 2x^2 + 1}{x(x^2+1)^2}\, dx$ is

   (A) $\ln\sqrt{3} - \frac{\pi}{24} + \frac{\sqrt{3}-2}{4}$; (B) $\ln\sqrt{3} - \frac{\pi}{12} + \frac{\sqrt{3}-2}{4}$;

   (C) $\ln\sqrt{3} - \frac{\pi}{24} + \frac{\sqrt{3}-2}{8}$; (D) $\ln\sqrt{3} - \frac{\pi}{12} + \frac{\sqrt{3}-2}{8}$.

5. The region bounded by curves $y = e^{-x}$, $y = 0$, $x = 0$ and $x = 1$ is rotated about the y-axis. Then the **volume** of the resulting solid of revolution is

   (A) $\frac{\pi}{2}(1 - e^{-2})$; (B) $2\pi(1 - 2e^{-1})$;

   (C) $2\pi(1 - e^{-1})$; (D) $\pi\big(\sqrt{2} + \ln(1 + \sqrt{2})\big)$.

6. The limit $\lim\limits_{x\to 0} \dfrac{\int_0^x \left(\int_0^{\sin t} \sqrt{1+u^2}\, du \right) dt}{\tan^2 x} =$

(A) 0; (B) $\frac{1}{2}$; (C) 1; (D) Does not exist.

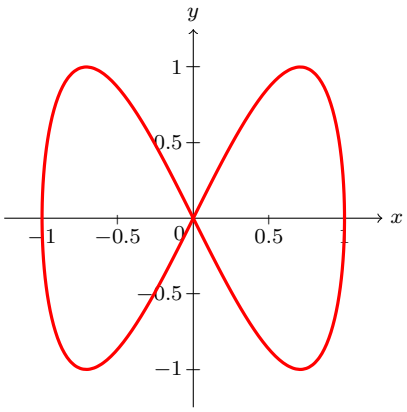
7. Which pair of parametric equations represents the graph below?

(A) $\begin{cases} x = \cos\theta \\ y = \theta + \sin\theta \end{cases}$;

(B) $\begin{cases} x = \sin(3\theta) \\ y = \cos(4\theta) \end{cases}$;

(C) $\begin{cases} x = \theta - \sin\theta \\ y = \cos\theta \end{cases}$;

(D) $\begin{cases} x = \sin\theta \\ y = \sin(2\theta) \end{cases}$.



8. The greatest integer function $[\![x]\!]$ is a function from $\mathbb{R}$ to $\mathbb{Z}$ with $x-1 < [\![x]\!] \le x$. The **value** of $\int_0^2 [\![x^2]\!]\, dx$ is

(A) $\frac{8}{3}$; (B) 1; (C) $7-\sqrt{2}-\sqrt{3}$; (D) $5-\sqrt{2}-\sqrt{3}$.

9. Let f be a **continuous** function on $\mathbb{R}$ satisfying $f(x) = x^{-5}\int_0^x \left(1-\cos(t^2)\right) dt$ for $x \neq 0$. Then $f(0)$ equals

(A) $\frac{1}{2}$; (B) $\frac{1}{5}$; (C) $\frac{1}{10}$; (D) $\frac{1}{20}$.

10. The base of a solid S is the region enclosed by curves $y = \sec x$, $y = \tan x$, $x = 0$ and $x = \pi/4$. The cross-sections perpendicular to the x-axis are **squares**. Then the **volume** of S is

(A) $4 - 2\sqrt{2} - \pi/2$; (B) $4 - \sqrt{2} - \pi/2$;

(C) $4 - 2\sqrt{2} - \pi/4$; (D) $4 - \sqrt{2} - \pi/4$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣。)

11. Which of the following statements are **true**?

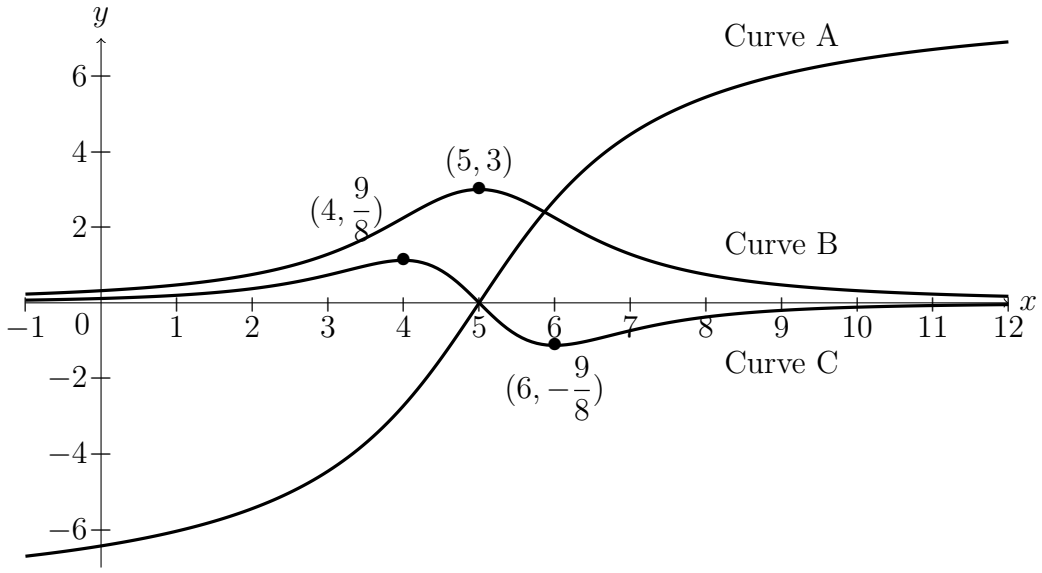
    (A) If both $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 0} g(x) = \infty$ hold, then it follows that $\lim_{x\to 0}\big(f(x) - g(x)\big) = 0$.

    (B) The equation $x^{10} - 10x^2 + 8 = 0$ has a root in $(0, 10)$.

    (C) If $|f|$ is integrable on $[0, 1]$, then so is f.

    (D) Every continuous function defined on $\mathbb{R}$ has at most two horizontal asymptotes.

12. The figure below shows graphs of f, f' and f''. Which of the following statements are **true**?



    (A) The curve B represents the graph of f.

    (B) f attains a local maximum at $x = 5$.

    (C) f is concave upward on $(0, 5)$.

    (D) The largest slope of the graph of f on $[0, 10]$ is happened at $x = 5$.

13. Which ones are **convergent**?

    (A) $\displaystyle\int_2^\infty \frac{1}{x(\ln x)^3}\,dx$; (B) $\displaystyle\int_0^{1/2} \frac{1}{x^2(\ln x)^4}\,dx$;

    (C) $\displaystyle\int_{-1}^1 \sqrt[3]{\frac{\sin x}{x^2}}\,dx$; (D) $\displaystyle\int_0^1 \frac{\sin\sqrt{x}}{x}\,dx$.

14. Which of the following statements are **true**?

    (A) If f is differentiable and increasing on $\mathbb{R}$, then $f(x) > 0$ for all $x \in \mathbb{R}$.

    (B) If $f(x)$ has a critical point at $x = c$, then $f'(c) = 0$.

    (C) If $f(t)$ is differentiable on $(0, \infty)$ and has a limit as $t \to \infty$, then $\lim_{t\to\infty} f'(t) = 0$.

    (D) A continuous function on $[a, b]$ attains its absolute maximum.

15. Consider the function $f(x) = \begin{cases} x\cos x, & \text{if } x \text{ is rational} \\ \sin x, & \text{if } x \text{ is irrational} \end{cases}$.
    Which of the following statements are **true**?

    (A) $f(x)$ is continuous at $x = 0$.

    (B) $f(x)$ is differentiable at $x = 0$.

    (C) $f(x)$ is continuous at infinite many points.

    (D) $f(x)$ is differentiable at infinite many points.

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣。)

1. The **length** of the parametric curve $x = \cos\theta$, $y = \theta + \sin\theta$, $\theta \in [0, \pi]$ is ____(1)____.

2. The equation of the **tangent line** to the curve $y\sin(2x) = x\cos(2y)$ at the point $(\pi/2, \pi/4)$ is $y = mx + b$. Then $(m, b) =$ ____(2)____.

3. Let $f(x)$ and $g(x)$ be polynomials of the **third** degree, and $f(x) - g(x) = x^3 + ax^2 + bx + c$, where a, b, c are real numbers. Assume that $f(x)$ and $g(x)$ are **tangent** to each other at $x = 1$. Moreover, $f(x)$ and $g(x)$ **only intersect** at $x = 1$. Then $(a, b, c) =$ ____(3)____.

4. The limit $\lim_{x\to 0} \dfrac{|6x - 1| - |6x + 1|}{x} =$ ____(4)____.

5. The line $y = mx$ cuts the region bounded above by the curve $y = x(1 - x)$ and below by the x-axis into two parts. Then, the areas of the two parts are **equal** when m is ____(5)____.