一百零七學年度第二學期微積分會考試題 (A 卷)

說明：

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。試卷加答案卷、答案卡共計 6 頁。

(3) 試卷包括選擇題與計算/證明，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ **Part1：單選擇題 (Single choice questions)**
(單選十題，每題五分，共五十分。)
(10 questions,each questions is worth 5 points,for 50 points in total.)

1. The integral of $\int_0^2 \int_{x/2}^1 \sin(y^2)dydx$ equals

   (A) $\sin 1$; (B) $\cos 1$; (C) $1 - \sin 1$; (D) $1 - \cos 1$.

2. The first three nonzero terms of the Maclaurin series of $f(x) = \ln \frac{1+x}{1-x}$ are

   (A) $2x + 2x^3/3! + 2x^5/5!$; (B) $2x + 2x^3/3 + 2x^5/5$;

   (C) $2x - 2x^3/3! + 2x^5/5!$; (D) $2x - 2x^3/3 + 2x^5/5$.

3. Which of the following is the tangent line to the parametric curve $\gamma(t) = \langle te^t, t - 2\ln t\rangle$ at $\gamma(1)$?

   (A) $ex + y = e^2 + e$; (B) $x + y = e + 1$;

   (C) $x + ey = 2e$; (D) $x + 2ey = 3e$.

4. Find the smallest value of α such that the inequality $(x + 2y + 3z)^4 \leq \alpha(x^4 + 2y^4 + 3z^4)$ holds for all real numbers x, y and z.

   (A) 6^3; (B) 5^3; (C) 4^3; (D) 3^3.

5. The maximum value of $x^2 + y^2$ over the circle $(x + 1)^2 + (y + 1)^2 = 4$ equals

   (A) $4 + 4\sqrt{2}$; (B) 10; (C) $5 + 4\sqrt{2}$; (D) $6 + 4\sqrt{2}$.

6. Let S be the portion of the hemisphere $f(x,y) = \sqrt{25 - x^2 - y^2}$ that lies above the disk $x^2 + y^2 \leq 9$. Then, the surface area of S equals

   (A) π;   (B) 10π;   (C) 5π;   (D) 12π.

7. Suppose $f(x,y)$ is a continuous function satisfying

$$\lim_{(x,y)\to(1,2)} \frac{f(x,y) - x^2 - y^2}{\sqrt{(x-1)^2 + (y-2)^2}} = 0.$$

   Then, $f_x(1,2)$ equals

   (A) 5;   (B) 4;   (C) 2;   (D) ∞.

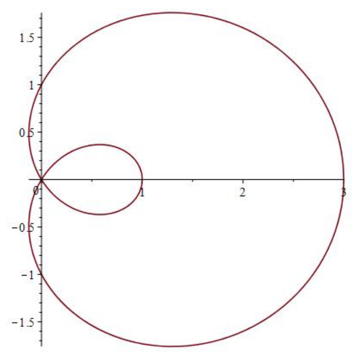
8. Let E be the solid bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + (y-1)^2 = 1$. Then, the volume of E is

   (A) $\frac{16}{9}(3\pi - 1)$;   (B) $\frac{16}{9}(3\pi - 2)$;   (C) $\frac{16}{9}(3\pi - 4)$;   (D) $\frac{16}{9}(3\pi - 8)$.

9. Let $T(u,v) = (2019u + 2020v, 2017u + 2018v)$ and R be the trapezoidal with vertices $(0,0)$, $(4,0)$, $(1,2)$ and $(3,2)$. Suppose S is a region such that $T(S) = R$. Then, the area of S equals

   (A) 3;   (B) 6;   (C) 12;   (D) 24.

10. Consider the polar curve $r = 1 + 2\cos\theta$ and let R be the region inside the larger loop but outside the smaller loop. Then, the area is



   (A) $\pi + \sqrt{3}$;   (B) $\pi + 2\sqrt{3}$;   (C) $\pi + 3\sqrt{3}$;   (D) $\pi + 4\sqrt{3}$.

◎ **Part2：多選擇題 (Multiple choice questions)**

(多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣。)

(5 questions,each questions is worth 5 points for 25 points in total. The correct answer is worth 5 points. Answers at a distance 1 from the correct answer are worth 3 points,other answers are worth no points.)

Example:

Answer: ABC

Student: ABC => 5 points , AB => 3 points, AD => 0 points, ABD => 0 points

11. Let $f(x) = \sum_{n=0}^{\infty} \frac{x^{4n}}{(2n)!}$. Which of the following falls in the **Range** of f?

    (A) $-1/2$; (B) $1/2$; (C) $3/2$; (D) $5/2$.

12. Assume that $\sum_{n=1}^{\infty} a_n x^n$ is convergent at $x = 1$ and divergent at $x = 2$. Which of the following statements are **False**?

    (A) $\lim_{n\to\infty} a_n = 0$.

    (B) $\sum_{n=1}^{\infty} a_n 2^{-n}$ is absolutely convergent.

    (C) $\sum_{n=1}^{\infty} a_n 3^n$ is convergent.

    (D) $\sum_{n=1}^{\infty} a_n n x^{n-1}$ is convergent at $x = 3$.

13. Assume that $f(x, y)$ is differentiable and set $g(t) = f(at, bt)$, where a and b are constants. Which of the following are **True**?

    (A) If, for any $a, b \in \mathbb{R}$, g has a local maximum at 0, then f has a local maximum at $(0, 0)$;

    (B) If, for any $a, b \in \mathbb{R}$, g has a local minimum at 0, then $\nabla f(0, 0) = \langle 0, 0\rangle$;

    (C) If $\nabla f(0, 0) = \langle 0, 0\rangle$, then g has a local minimum at 0 for any $a, b \in \mathbb{R}$;

    (D) If f has a local maximum at $(0, 0)$, then g has a local maximum at 0 for any $a, b \in \mathbb{R}$.

14. Let $f(x, y, z) = e^x - y^4 - z^3 + xyz$ and $g(t) = f(at, 1 + bt, 2 + ct)$. Suppose there is $\epsilon > 0$ such that g is decreasing on $(0, \epsilon)$. Then, (a, b, c) can be

    (A) $(-3, 4, 12)$; (B) $(3, -4, -12)$; (C) $(0, 3, -1)$; (D) $(0, -3, 1)$.

15. Let f be the following function.

$$f(t,s)=\int_{-t}^{t}\int_{-s\sqrt{1-(x/t)^2}}^{s\sqrt{1-(x/t)^2}}(x^2+y^2)dydx.$$

Which of the followings are **True**?

(A) $f(1,1)=\pi/2$; (B) $f_t(2,1)=3\pi$;

(C) $f_s(3,1)=7\pi$; (D) $f_{ts}(4,1)=12\pi$.

◎Part3：計算/證明題 (Questions of calculations and proofs)
(Answer the problems as thoroughly as possible. Be sure to include all your work. Partial credit will be given even if the answer is not fully correct.)

1. Let $f(x,y)=\frac{1}{x}+\frac{1}{y}$ for $x\neq 0$ and $y\neq 0$.

   (A) (9 points) Assume that, under the constraint $\frac{1}{x^2}+\frac{1}{y^2}=1$, f attains its maximum at P. Find P.

   (B) (3 points) Let P be the point in part (A). Assume that, on the parametric curve $\langle t, at^2+bt\rangle$, f has a local minimum at P. Find the values of a,b.

2. (A) (3 points) Determine the convergency of the sequence $\{|a_n|^{1/n}\}_{n=1}^{\infty}$, where

$$a_n=\begin{cases}2^{-n+\sqrt{n}}, & \text{if } n \text{ is even;}\\ 3^{-n+\sqrt{n}}, & \text{if } n \text{ is odd.}\end{cases}$$

   (B) (7 points) Prove the following claim: If there are some real $0<q<1$ and integer $N\geq 0$ such that

$$|b_n|^{1/n}\leq q,\quad \forall n\geq N,$$

   then $\sum_{n=1}^{\infty} b_n$ is convergent.

   (C) (3 points) Let $\{a_n\}_{n=1}^{\infty}$ be the sequence in part (A). Does part (B) imply that $\sum_{n=1}^{\infty} a_n$ is convergent?