一百零八學年度第二學期微積分會考試題 (A 卷)

說明：

(1) 答題之前請先檢查所取得之試卷與答案卷、卡卷別是否相符。

(2) 測驗時間 120 分鐘。試卷加答案卷、答案卡共計 7 頁。

(3) 試卷包括選擇題與填空題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ **Part1：單選擇題**
(Multiple-Choice Questions, Each Problem with Single Correct Answer)
(單選十題，每題五分，共五十分。)
(10 questions, each question is worth 5 points, for 50 points in total.)

1. If the length of the polar curve $r = 3\theta^2$ with $0 \le \theta \le 2\pi$ is $a[(\pi^2+1)^{3/2} - 1]$, then a is equal to

   (A) 2. (B) 4. (C) 8. (D) 16.

2. Which one is the Maclaurin series of $\cos^2 x$?

   (A) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$.

   (B) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{(2n)!}$.

   (C) $\frac{1}{2} + \frac{1}{2}\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{(2n)!}$.

   (D) $\frac{1}{2} + \frac{1}{2}\sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n}}{(2n)!}$.

3. Let $w = \ln(x^2 + y^2 + z^2)$ and

$$x = ue^v \sin u, \quad y = ue^v \cos u, \quad z = ue^v$$

   What is the value of $\frac{\partial w}{\partial v}$ at $(u, v) = (6\pi^2, 7)$?

   (A) 2. (B) 4. (C) 6. (D) $48\pi^2 e^7$.

4. What is the value of the following iterated integral?

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{(1+x^2+y^2)^2}dxdy$$

(A) $\frac{1}{2}$. (B) π. (C) 2π. (D) ∞.

5. Consider the following function

$$f(0,0)=0, \quad f(x,y)=\frac{xy(x^2-y^2)}{x^2+y^2} \quad \text{for } (x,y)\neq(0,0).$$

Set $g=f_x$. Then, the directional derivative of g at $(0,0)$ in the direction of $\mathbf{u}=\langle\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\rangle$ is

(A) 0. (B) $\frac{\sqrt{2}}{2}$. (C) $-\frac{\sqrt{2}}{2}$. (D) $\sqrt{2}$.

6. Let f be a function of two variables with continuous partial derivatives. Assume that the directional derivatives of f at $(0,0)$ in the direction of $\langle 3,4\rangle$ and $\langle 4,-3\rangle$ are respectively 3 and 4. Then, the directional derivative of f at $(0,0)$ in the direction of $\langle 1,1\rangle$ is

(A) 5. (B) -5. (C) $\frac{5\sqrt{2}}{2}$. (D) $-\frac{5\sqrt{2}}{2}$.

7. Assume that there is a number $\delta>0$ such that

$$\sum_{n=0}^{\infty}2^{n+1}a_{n+2}x^{n+1}+\sum_{n=1}^{\infty}\frac{2^n}{a_n}x^n=1-a_1+\sum_{n=1}^{\infty}3\cdot 2^n x^n, \quad \text{for all } |x|<\delta.$$

Then, $\lim_{n\to\infty}a_n$ is equal to

(A) $\frac{1}{2}$. (B) 1. (C) $\frac{3+\sqrt{5}}{2}$. (D) $\frac{1+\sqrt{3}}{2}$.

8. Let $f(x,y)$ be a continuous function defined for $-\infty<x,y<\infty$ and satisfying

$$\lim_{(x,y)\to(e,\pi)}\frac{f(x,y)-1-2(x-e)-3(y-\pi)}{\sqrt{(x-e)^2+(y-\pi)^2}}=0.$$

Then, $f(e,\pi)+f_x(e,\pi)+f_y(e,\pi)$ is equal to

(A) $e\pi$. (B) 2. (C) 3. (D) 6.

9. Let $E = \{-\frac{1}{3}, \frac{1}{4}, 1, 2\}$. How many points $x \in E$ at which the power series $\sum_{n=1}^{\infty} (\sin n)(2x-1)^n$ converges?

   (A) 1. (B) 2. (C) 3. (D) 4.

10. For $t > 0$, let $A(t)$ be the area of the surface $f(x, y) = xy\cos(txy)$ with $x^2+y^2 \leq 3$. Then, $\lim\limits_{t \to 0^+} A(t)$ is equal to

   (A) $\frac{7\pi}{3}$. (B) $\frac{14\pi}{3}$. (C) $\frac{21\pi}{3}$. (D) $\frac{28\pi}{3}$.

◎ **Part2：多選擇題**

(Multiple-Choice Questions with More Than One Correct Answers)

(多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣。)

(5 questions, each question is worth 5 points for 25 points in total. The correct answer is worth 5 points. Answers at a distance 1 from the correct answer are worth 3 points, other answers are worth no points.)

Example:

Answer: ABC

Student: ABC => 5 points , AB => 3 points, AD => 0 points, ABD => 0 points

11. Consider the following polar equations.

$$\gamma_1 : r^2 = \sin(2\theta), \quad \gamma_2 : r^2 = \cos(2\theta), \quad \gamma_3 : r = 1 + \sin\theta, \quad \gamma_4 : r = -1 + \sin\theta.$$

   Which of the following statements are **TRUE**?

   (A) There are three horizontal tangent lines on the graph of γ_1.

   (B) The graph of γ_2 is symmetric about the x-axis and the y-axis.

   (C) The graphs of γ_3 and γ_4 are different.

   (D) The area of the region enclosed by the graph of γ_1 is the same as the area of the region enclosed by the graph of γ_2.

12. Which of the following series are convergent?

   (A) $\sum\limits_{n=1}^{\infty} n \tan\frac{1}{n}$. (B) $\sum\limits_{n=1}^{\infty} ne^{-n^2}$. (C) $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$. (D) $\sum\limits_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$.

13. Let f be a function defined by

$$f(0,0)=0, \quad f(x,y)=\frac{4xy^2}{4x^2+y^4} \quad \text{for } (x,y)\neq(0,0).$$

Which of the following statements are **TRUE**?

(A) f is continuous at $(0,0)$.

(B) $D_{\mathbf{u}}f(0,0)$ exists for any unit vector $\mathbf{u}$.

(C) $(0,0)$ is a critical point of f.

(D) f_x and f_y are continuous at $(0,0)$.

14. Which of the following statements are **TRUE**?

(A) $\displaystyle\int_3^6\int_1^3 \frac{x-1}{y-2}\sin(x-y)dxdy = \int_1^3\int_3^6 \frac{x-1}{y-2}\sin(x-y)dydx.$

(B) $\displaystyle\int_0^1\int_0^x \sqrt{x^2+y}dydx = \int_0^x\int_0^1 \sqrt{x^2+y}dxdy$ for all x.

(C) $\displaystyle\int_{-2}^2\int_0^5 e^{-2x^2-2y^2}\sin y\cos x dxdy = 0.$

(D) $\displaystyle\int_0^1\int_{\sqrt{x}}^1\int_0^{1-y} e^{-z^2}dzdydx = \int_0^1\int_0^{1-z}\int_0^{y^2} e^{-z^2}dxdydz.$

15. Subject to which constraint, the function $f(x,y,z)=x^2+y^2-z^2$ does **NOT** attain its maximum value?

(A) $x+y+z=1$.

(B) $x^2+y^2+z^2=1$.

(C) $e^{xz}+y^2=1$.

(D) $e^{x^3+z^3+x+z}=y^2$.

◎ **Part3：填空題 (Fill-in-the-blank)**
(填空五題，每題五分，共二十五分。)
(5 questions, each question is worth 5 points, for 25 points in total.)

1. Let $f(x,y)=\displaystyle\int_{x-y}^{x+y} e^{t^2}dt$. Then, $f_{xy}(1,1)=$ ____(1)____.

2. Let α and β be the extremum values of $f(x,y,z)=xy+z^2$ subject to the constraints $x-y=0$ and $x^2+y^2+z^2\leq 4$. Then, $\alpha+\beta=$ ____(2)____.

3. The volume of the solid lying below the cone $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = z$ is $\underline{\quad (3) \quad}$.

4. Let R be the region on the xy-plane bounded by the square with vertices $(0, 1), (1, 2), (2, 1)$ and $(1, 0)$. If $\iint_R (x + y)^2 \sin^2(x - y) dA = \frac{a}{6}(2 - \sin 2)$, then $a = \underline{\quad (4) \quad}$.

5. Let S be the surface obtained by rotating the curve $y = x^2 + 1$ on the xy-plane about the x-axis. Then, the tangent plane to S at $(1, -1, \sqrt{3})$ is $\underline{\quad (5) \quad}$.