《一百零九學年度第一學期微積分會考答案卷》(A 卷)

姓名					老師			
學號					系別			系

總分(第一部份~第三部份)	初閱		複閱	

第一、二、三部份合計總分	

第一部份：單選擇題

1	**A**	**2**	**C**	**3**	**C**	**4**	**D**	**5**	**B**
6	**D**	**7**	**A**	**8**	**A**	**9**	**C**	**10**	**B**

初閱		評分
複閱		

第二部份：複選擇題

11	**AC**	**12**	**AC**	**13**	**CD**	**14**	**ABD**	**15**	**AD**

初閱		評分
複閱		

Problem 1. (10%)

Set

(A) [Way I.]

$$\frac{1}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}.$$

$$1 = A(x+1)(x+2) + Bx(x+2) + Cx(x+1)$$

close $x = 0 \Rightarrow 1 = A \cdot 1 \cdot 2 \Rightarrow A = \frac{1}{2}$

$x = -1 \Rightarrow 1 = B(-1)(1) \Rightarrow B = -1$

$x = -2 \Rightarrow 1 = C(-2)(-1) \Rightarrow C = \frac{1}{2}$

[Way II.]

$$\frac{1}{x(x+1)(x+2)} = \frac{A}{x(x+1)} + \frac{B}{(x+1)(x+2)}.$$

$$= \frac{1}{2}\left(\frac{1}{x} + \frac{-1}{x+1}\right) - \frac{1}{2}\left(\frac{1}{x+1} + \frac{-1}{x+2}\right).$$

[common part]

$$\Rightarrow \frac{\frac{1}{2}}{x} + \frac{-1}{x+1} + \frac{\frac{1}{2}}{x+2}$$

$$\int \frac{1}{x(x+1)(x+2)}dx = \int \left(\frac{\frac{1}{2}}{x} + \frac{-1}{x+1} + \frac{\frac{1}{2}}{x+2}\right)dx$$

$$= \frac{1}{2}\ln|x| - \ln|x+1| + \frac{1}{2}\ln|x+2| + C$$

$$= \frac{1}{2}\ln\frac{|x(x+2)|}{(x+1)^2} + C$$

Note that:

If students do not write down the constant C, only one point will be given.

(B) $$\int_1^\infty \frac{dx}{x(x+1)(x+2)} = \lim_{t\to\infty}\int_1^t \frac{dx}{x(x+1)(x+2)}$$

From (A) $$= \lim_{t\to\infty} \frac{1}{2}\frac{|x(x+2)|}{(x+1)^2}\Big|_{x=1}^{x=t}$$

$$= \lim_{t\to\infty} \frac{1}{2}\left(\ln\frac{|t(t+2)|}{(t+1)^2} - \ln\frac{3}{2^2}\right)$$

$$\lim_{t\to\infty}\frac{|t(t+2)|}{(t+1)^2} = \lim_{t\to 0}\frac{|1(1+\frac{2}{t})|}{(1+\frac{1}{t})^2} = 1$$

Since $\ln(x)$ is continuous at $x = 1$

$$\Rightarrow \lim_{t\to 0}\frac{1}{2}\left(\ln\frac{|t(t+2)|}{(t+1)^2} - \ln\frac{3}{4}\right)$$

$$= \frac{1}{2}\left(\ln\lim_{t\to\infty}\frac{|t(t+2)|}{(t+1)^2} - \ln\frac{3}{4}\right)$$

$$= -\frac{1}{2}\ln\frac{3}{4} = \frac{1}{2}\ln\frac{4}{3} = \frac{1}{2}(\ln 4 - \ln 3) = \frac{1}{2}(2\ln 2 - \ln 3) = \ln 2 - \frac{1}{2}\ln 3$$

(C) $$I_n = \frac{1}{n!}\sum_{k=0}^{n}(-1)^{k+1}\binom{n}{k}\ln(1+k) \quad \text{or} \quad \frac{1}{n!}\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}\ln(1+k)$$

$$\text{or} \quad \frac{1}{n!}\sum_{k=2}^{n+1}(-1)^k\binom{n}{k-1}\ln k \quad \text{or} \quad \frac{1}{n!}\sum_{k=1}^{n+1}(-1)^k\binom{n}{k-1}\ln k$$

$$\text{or} \quad \frac{1}{n!}\sum_{k=2}^{n+1}(-1)^k\binom{n}{k-1}\ln k \quad \text{or} \quad \frac{1}{n!}\sum_{k=1}^{n+1}(-1)^k\binom{n}{k-1}\ln k$$

Problem 2. (15%) Let $f(x) = \dfrac{2x^3}{x^2-4}$.

Sol. Since the domain of f is $\mathbb{R}\backslash\{-2,2\}$, so $x = \pm 2$ can't be critical numbers or inflection points.

(A) (i) (1.5pts) Since

$$\lim_{x\to 2^+} f(x) = \infty \quad \text{and} \quad \lim_{x\to(-2)^+} f(x) = \infty\,,$$

so f has vertical asymptotes $x = -2$ and $x = 2$.

(ii) (1.5pts) Evaluate that $f(x) = \dfrac{2x^3}{x^2-4} = 2x + \dfrac{8x}{x^2-4}$. Since

$$\lim_{x\to\infty}\{f(x) - 2x\} = \lim_{x\to\infty}\{\frac{8x}{x^2-4}\} = 0 = \lim_{x\to-\infty}\{f(x)-2x\}\,,$$

so f has a slant asymptote $y = 2x$.

(B) Since $f(x) = \dfrac{2x^3}{x^2-4}$, so

$$f'(x) = \frac{6x^2(x^2-4) - 2x^3\cdot 2x}{(x^2-4)^2} = \frac{2x^2(x^2-12)}{(x^2-4)^2}.$$

(i) Let $f'(x) = 0$. Then $x = \pm 2\sqrt{3}$. We obtain critical numbers are $x = \pm 2\sqrt{3}$.

(ii) Since $f'(x) < 0$ on $(-2\sqrt{3},-2) \cup (-2,2) \cup (2,2\sqrt{3})$, f is decreasing on $(-2\sqrt{3},-2) \cup (-2,2) \cup (2,2\sqrt{3})$.

(iii) Since $f'(x) > 0$ on $(-\infty,-2\sqrt{3}) \cup (2\sqrt{3},\ \infty)$, f is increasing on $(-\infty,-2\sqrt{3}) \cup (2\sqrt{3},\infty)$.

(iv) Moreover, by the First Derivative Test, f has a local minimum value $f(2\sqrt{3}) = 6\sqrt{3}$ and a local maximum value $f(-2\sqrt{3}) = -6\sqrt{3}$.

(C) Evaluate that

$$\begin{aligned}
f''(x) &= \frac{(8x^3-48x)(x^2-4)^2 - (2x^4-24x^2)\cdot 2(x^2-4)\cdot 2x}{(x^2-4)^4} \\
&= \frac{4x(2x^2-12)(x^2-4) - (2x^4-24x^2)\cdot 4x}{(x^2-4)^3} \\
&= \frac{4x(2x^4-20x^2+48-2x^4+24x^2)\cdot 4x}{(x^2-4)^3} \\
&= \frac{16x(x^2+12)}{(x^2-4)^3}.
\end{aligned}$$

(i) Let $f''(x) = 0$. Then $x = 0$.

(ii) Since $f''(x) < 0$ on $(0,2)$ and $f''(x) > 0$ on $(-2,0)$, f has an inflection point $(0,0)$.

(iii) Since $f''(x) > 0$ on $(-2,0) \cup (2,\infty)$, f is concave upward on $(-2,0) \cup (2,\infty)$.

(iv) Since $f''(x) < 0$ on $(-\infty,-2) \cup (0,2)$, f is concave downward on $(-\infty,-2) \cup (0,2)$.

□

(D) Sketch the graph of f.