

《一百一十二學年度第二學期微積分會考答案卷》(A 卷)

第一部份：單選擇題

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|----|---|
| 1 | B | 2 | C | 3 | A | 4 | D | 5 | A |
| 6 | A | 7 | D | 8 | B | 9 | C | 10 | B |

第二部份：複選擇題

| | | | | | | | | | |
|----|-----|----|------|----|---------------|----|----|----|-----|
| 11 | ABC | 12 | ABCD | 13 | ABD， AD 皆對 | 14 | AC | 15 | BCD |
|----|-----|----|------|----|---------------|----|----|----|-----|

第三部份：填充題

| | |
|---|---|
| A | $(0, 0), (\pm\sqrt{2}/8, \pm\sqrt{2}/4).$ |
| B | $(\pm\sqrt{2}/8, \pm\sqrt{2}/4).$ |
| C | $(3 + \sqrt{2}, 3 - \sqrt{2}).$ |

《一百一十二學年度第二學期微積分會考答案卷》(B卷)

第一部份：單選擇題

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|---|---|---|---|---|---|---|---|----|---|
| 1 | B | 2 | A | 3 | C | 4 | A | 5 | D |
| 6 | B | 7 | D | 8 | A | 9 | C | 10 | B |

第二部份：複選擇題

| | | | | | | | | | |
|----|-----|----|---------------|----|------|----|-----|----|----|
| 11 | ABC | 12 | ABD， AD 皆對 | 13 | ABCD | 14 | BCD | 15 | AC |
|----|-----|----|---------------|----|------|----|-----|----|----|

第三部份：填充題

| | |
|---|---|
| A | $(0, 0), (\pm\sqrt{2}/8, \pm\sqrt{2}/4).$ |
| B | $(\pm\sqrt{2}/8, \pm\sqrt{2}/4).$ |
| C | $(3 + \sqrt{2}, 3 - \sqrt{2}).$ |

NYCU Calculus Exam: Spring, 2024

1. SOLUTION TO PART (A) PROBLEM 2

Step 1.

The spherical coordinate system is given by :

$$\begin{aligned}x &= \rho \sin \varphi \cos \theta, \\y &= \rho \sin \varphi \sin \theta, \\z &= \rho \cos \varphi\end{aligned}\tag{1.1}$$

Grading suggestion: (1.1) should be stated correctly. This worth 1 point. Otherwise, give 0 point .

Step 2. Based on (1.1), we derive

$$\begin{aligned}x^2 + y^2 + (z - 5)^2 &= \rho^2 - 10\rho \cos \varphi + 25, \\z - 5 &= \rho \cos \varphi - 5,\end{aligned}\tag{1.2}$$

from which we get

$$\frac{z - 5}{(x^2 + y^2 + (z - 5)^2)^{\frac{3}{2}}} = \frac{\rho \cos \varphi - 5}{(\rho^2 - 10\rho \cos \varphi + 25)^{\frac{3}{2}}}\tag{1.3}$$

Grading suggestion: If a student writes down either one of (1.2) or (1.3), give 1 point. Otherwise, give 0 point .

Step 3

The Jacobian for the spherical coordinate transformation is $\rho^2 \sin \varphi$.

Grading suggestion: This worth 1 point. Or equivalently, the student should state $dV = \rho^2 \sin \varphi d\rho d\varphi d\theta$.

Step 4 So, it follows that

$$\begin{aligned}I &= \int_0^{2\pi} \int_0^\pi \int_0^3 \frac{\rho \cos \varphi - 5}{(\rho^2 - 10\rho \cos \varphi + 25)^{\frac{3}{2}}} \rho^2 \sin \varphi d\rho d\varphi d\theta \\&= 2\pi \int_0^3 \rho^2 \int_0^\pi f(\rho, \varphi) d\varphi d\rho,\end{aligned}\tag{1.4}$$

where

$$f(\rho, \varphi) = \frac{(\rho \cos \varphi - 5) \sin \varphi}{(\rho^2 - 10\rho \cos \varphi + 25)^{\frac{3}{2}}}\tag{1.5}$$

Grading suggestion: As long as a student writes out the **the first line** of (1.4) correctly, we give 2 points here. Otherwise, we give 0 point .

2. SOLUTION TO PART (B) PROBLEM 2

Step 5

Recall

$$G(\rho) = \int_0^\pi \frac{(\rho \cos \varphi - 5) \sin \varphi}{(\rho^2 - 10\rho \cos \varphi + 25)^{\frac{3}{2}}} d\varphi.$$

By using the substitution as provided by the Hint, and through some computations, we get

$$G(\rho) = \frac{1}{50\rho} \int_{5-\rho}^{5+\rho} \frac{\rho^2 - t^2 - 25}{t^2} dt. \quad (2.1)$$

Grading suggestion: (2.1) has to be stated correctly to get 2 points. Otherwise, give 0 .

Step 6

Through some computations, we get from (2.1) that

$$G(\rho) = -\frac{2}{25}. \quad (2.2)$$

Grading suggestion: A student get 1 point for this step if and only if both (2.1) and (2.2) are written out correctly. Otherwise, give 0 point.

3. SOLUTION TO PART (C) PROBLEM 2

Based on (2.2), we get:

$$I = 2\pi \int_0^3 \rho^2 \left(\frac{-2}{25}\right) d\rho = \frac{-36\pi}{25}. \quad (3.1)$$

Grading suggestion: 2 points will be given if all of (2.1) , (2.2) , (3.1) are stated correctly. In particular, if the answer $\frac{-36\pi}{25}$ is wrong, we still give 0 point .