九十四學年度第一學期微積分會考試題 甲卷

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。甲乙兩份試卷加答案卷共計 6 頁。

(3) 甲卷爲一般試卷，包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。乙卷爲挑戰題試卷，可自行決定是否作答，計 40 分，不佔學期成績。甲乙兩卷成績合計後，將做爲微積分獎給獎依據或教師加分參考。

(4) 乙卷採「延時加考」之方式進行，於測驗時間 110 分鐘結束，並回收甲卷後，再額外提供 30 分鐘時間作答乙卷。

(5) 請先確實填入相關個人資料。答題時請依題號空格作答，否則不予計分。

(6) 題目將於一月十五日於網站公佈。

◎ 選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. Under ideal conditions a certain bacteria population is known to double every three hours. Suppose that there are initially 100 bacteria. When will the population first reach 50,000 ? (提示： $\ln 2 \approx 0.6931$, $\ln 3 \approx 1.0986$, $\ln 5 \approx 1.6094$)

   A) 26 hours; B) 27 hours; C) 28 hours; D) 29 hours.

2. $\lim_{x\to 0}\left(x-\frac{1}{x}\right)\sin x =$

   A) –1; B) 0; C) 1; D) not exist.

3. Let f be a function from $\mathbb{R}$ to $\mathbb{R}$. Which of the following statements is true ?

   A) If $\lim_{x\to\infty}(f(x+1)-f(x))=0$, then $\lim_{x\to\infty} f(x)$ exists and is finite;

   B) $\lim_{x\to 0} f(x)=\lim_{x\to 0} f(x^3)$;

   C) $\lim_{x\to 0} f(x)=\lim_{x\to 0} f(x^2)$;

   D) If $\lim_{x\to\infty} f(x)$ exists, then $\lim_{x\to\infty} f'(x)$ exists.

   本題的題目應做適當修正：

   B) If $\lim_{x\to 0} f(x^3)$ exists, then $\lim_{x\to 0} f(x)$ exists and $\lim_{x\to 0} f(x)=\lim_{x\to 0} f(x^3)$;

   C) If $\lim_{x\to 0} f(x^2)$ exists, then $\lim_{x\to 0} f(x)$ exists and $\lim_{x\to 0} f(x)=\lim_{x\to 0} f(x^2)$;

4. Let

$$f(x)=\begin{cases} x, & \text{if } x \text{ is a rational number,} \\ 1-x, & \text{if } x \text{ is an irrational number.} \end{cases}$$

On what values of x is f continuous ?

A) any rational numbers; B) any irrational numbers; C) no such x; D) $x=1/2$.

5. Let $a>0$. Consider the function $f(x)=\dfrac{1}{1+|x|}+\dfrac{1}{1+|x-a|}$. Which of the following statements is **NOT** true ?

A) The global maximum of f exists;

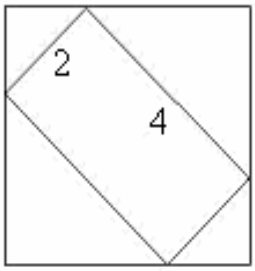
B) f has no global minimum;

C) f has local maxima $\dfrac{a+2}{a+1}$ at $x=0$ and at $x=a$;

D) f has no local minimum.

6. Find the maximum area of a rectangle that can be circumscribed about a given rectangle with length 2 and width 4 (See the figure below).

A) 15; B) 16; C) 17; D) 18.



7. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 ft/s, how fast is the angle between the top of the ladder and the wall changing when the angle is $\dfrac{\pi}{4}$ rad ?

A) $\dfrac{\sqrt{2}}{5}$ rad/s ; B) $\dfrac{\sqrt{3}}{5}$ rad/s ; C) $\dfrac{\sqrt{2}}{8}$ rad/s ; D) $\dfrac{\sqrt{3}}{8}$ rad/s .

8. How many zeros does the function $f(x)=2^x-1-x^2$ have on the real line ?

A) 2; B) 3; C) 4; D) 5.

9. Evaluate $\int_0^2 \dfrac{1}{(x-1)^2}\,dx.$

A) 0; B) 1; C) 2; D) diverges.

10. Find $\lim_{x\to 0}\frac{1}{x^3}\int_0^x \frac{t^2}{1+t^4}dt.$

A) 1/2; B) 1/3; C) 1/4; D) 1/5.

◎ 填空題 (十題，每題五分，共五十分，答錯不倒扣)

1. $\lim_{x\to 0}\left(\frac{3^x+5^x}{2}\right)^{1/x}$ = ____(1)____.

2. If $\lim_{x\to 1}\frac{f(e^{2x}+x^{2x}-e^2)-f(1)}{2x-2}=4$, then $f'(1)=$ ____(2)____.

3. Find y' while $x^y=y^x$. ____(3)____

4. $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}(\ln(n+k)-\ln(n))=$ ____(4)____.

5. Let $f'(x)=\frac{\sin x}{x}$, $f(0)=1$, $f(\pi)=3$, then $\int_0^{\pi} f(u)du=$ ____(5)____.

本題的題目應做適當修正：

Let $f'(x)=\frac{\sin x}{x}$, $f(0)=$ constant A, $f(\pi)=3$, then $\int_0^{\pi} f(u)du=$ ____(5)____.

6. $\int_0^{(\ln 3)/2}\frac{1}{1+e^{2x}}dx=$ ____(6)____.

7. Find $\int\frac{3+x}{\sqrt{13+4x+x^2}}dx.$ ____(7)____

8. Calculate $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx.$ (You might want to use substitution $u=\pi-x$.) ____(8)____

9. Find the volume common to two spheres, each with radius r, if the center of each sphere lies on the surface of the other sphere. ____(9)____

10. Find the exact length of the polar curve $r=\theta^2$ for $0\le\theta\le 2\pi$. ____(10)____