九十四學年度第二學期微積分會考試題 甲卷

說明:
(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。
(2) 測驗時間 110 分鐘。甲乙兩份試卷加答案卷共計 6 頁。
(3) 甲卷爲一般試卷，包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。乙卷爲挑戰題試卷，可自行決定是否作答，計 40 分，不佔學期成績。甲乙兩卷成績合計後，將做爲微積分獎給獎依據或教師加分參考。
(4) 乙卷採「延時加考」之方式進行，於測驗時間 110 分鐘結束，並回收甲卷後，再額外提供 30 分鐘時間作答乙卷。
(5) 請先確實填入相關個人資料。答題時請依題號空格作答，否則不予計分。
(6) 題目將於六月二十五日於網站公佈。

◎ 選擇題 **(單選十題，每題五分，共五十分，答錯不倒扣)**

1. The Sierpinski carpet is constructed by removing the center one-ninth of a square of side 1, then removing the centers of the eight smaller remaining squares, and so on. (Figure 1 shows the first three steps of the construction.) Let L be the intersection of the Sierpinski carpet and a vertical segment of length 1 on the square from the top to the bottom. Which statement must be correct?

   A) L is empty;

   B) L has length l for some $0<l<1$;

   C) L has length 0;

   D) L has length 1;

   E) L is nonempty.

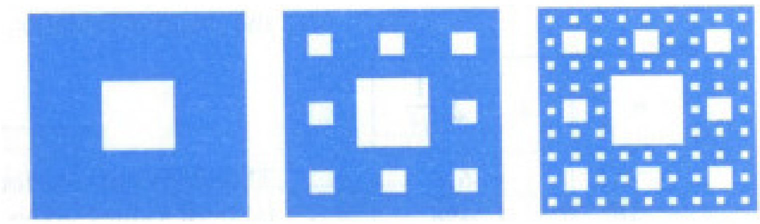


Figure 1: The first three steps of the construction

2. Which of the following vectors is not orthogonal to the vector $3\mathrm{i}+4\mathrm{j}+5\mathrm{k}$?
   A) 0; B) $3\mathrm{i}+4\mathrm{j}-5\mathrm{k}$; C) $15\mathrm{i}-9\mathrm{k}$; D) $6\mathrm{i}-8\mathrm{j}+10\mathrm{k}$.

3. Let $f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n^2}$. Let I, J, and K be the intervals of convergence for f, f', and f'', respectively. Which one contains only one end point?

A) I ;   B) J ;   C) K ;   D) none.

4. Let

$$f(x,y)=\begin{cases}\dfrac{xy^3-x^3y}{x^2+y^2}, & \text{if } (x,y)\neq(0,0),\\ 0, & \text{if } (x,y)=(0,0).\end{cases}$$

Then

A) $f_{xy}(0,0)$ does not exist;   B) $f_{xy}(0,0)=-1$;   C) $f_{xy}(0,0)=1$;   D) $f_{xy}(0,0)=0$.

5. Which of the following limits exists?

A) $\lim\limits_{(x,y)\to(0,0)}\dfrac{x^2}{x^2+y^2}$;   B) $\lim\limits_{(x,y)\to(0,0)}\dfrac{xy\cos y}{3x^2+y^2}$;   C) $\lim\limits_{(x,y)\to(0,0)}\dfrac{xy}{\sqrt{x^2+y^2}}$;

D) $\lim\limits_{(x,y)\to(0,0)}\dfrac{2x^2y}{x^4+y^2}$.

6. Let $z=f(x,y)$, where f is differentiable, $y=h(x)$, $x=g(t)$, $g(3)=3$, $g'(3)=5$, $h(3)=7$, $h'(3)=4$, $f_x(3,7)=6$, and $f_y(3,7)=-8$, then $\left.\dfrac{dz}{dt}\right|_{t=3}=$

A) -130;   B) -2;   C) -26;   D) 62.

7. $\int_0^1\int_0^{\sqrt{1-x^2}} xy\,dy\,dx=$

A) $1/16$;   B) $1/8$;   C) $1/4$;   D) $1/2$.

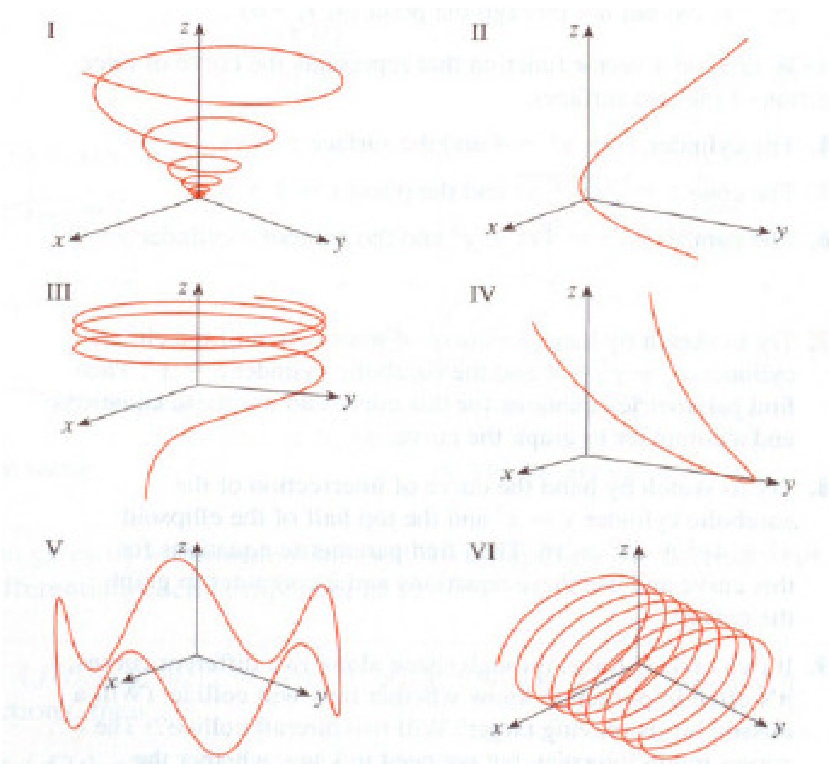
8. $\int_0^2\int_0^{\sqrt{y}} \exp(-x^5-y^4)\,dx\,dy=$

A) $\int_0^4\int_{x^2}^4 \exp(-x^5-y^4)\,dy\,dx$;   B) $\int_0^2\int_{\sqrt{x}}^{\sqrt{2}} \exp(-x^5-y^4)\,dy\,dx$;

C) $\int_0^{\sqrt{2}}\int_{x^2}^2 \exp(-x^5-y^4)\,dy\,dx$;   D) $\int_0^2\int_{x^2}^2 \exp(-x^5-y^4)\,dy\,dx$.

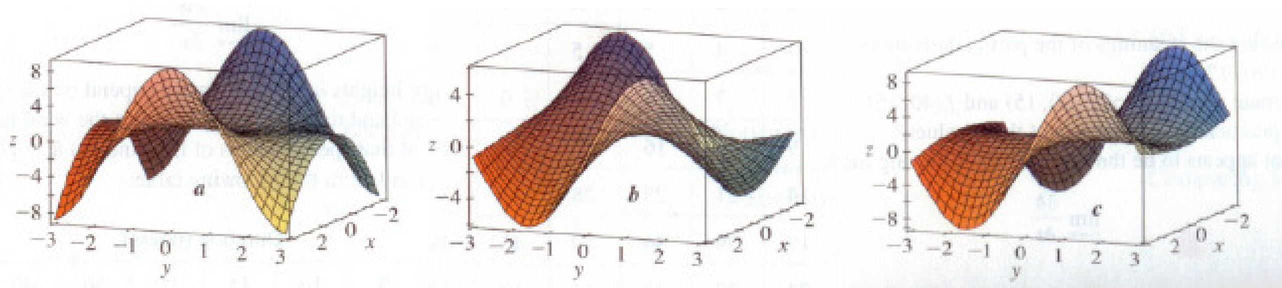
9. Some of graphs in Figure 2 labeled (I-VI) are graphs of the parametric equations (x,y,z) labeled (a-d): a. $(\cos t,\sin t,\sin 5t)$, b. $(\cos t,\sin t,\ln t)$, c. $(e^{-t}\cos 10t, e^{-t}\sin 10t, e^{-t})$, and d. $(\cos 4t, t, \sin 4t)$. Which statement is correct?

A) a's graph is I and b's graph is II;
B) b's graph is III and c's graph is IV;
C) c's graph is I and d's graph is VI;
D) d's graph is III and a's graph is V.



10. Surfaces in Figure 3, labeled by a, b, c, are graphs of a function f and its partial derivatives f_x and f_y. The ordering of the graphs of f, f_x, f_y is

A) a, b, c;
B) b, a, c;
C) c, a, b;
D) c, b, a.



◎ 填空題 (十題，每題五分，共五十分，答錯不倒扣)

1. Let (a_n) be a sequence of real numbers given by $a_1 = -2$, $a_{n+1} = \sqrt{6 + a_n}$, then $\lim_{n\to\infty} a_n$ = ____(1)____.

2. The interval of convergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n2^n}$ is ____(2)____.

3. Let $E=\left\{(x,y,z)\in \mathbb{R}^3 \mid 0\le y\le 1, -\sqrt{1-y^2}\le x\le \sqrt{1-y^2}, 0\le z\le \sqrt{y}\right\}$. Then $\iiint_E z\,dV =$ ____(3)____.

4. The directional derivative of $f(x,y,z)=xy^2z^3$ at (2,1,1) in the direction of the vector v = i + 2j + 2k is ____(4)____.

5. Let $E=\left\{(x,y,z)\in \mathbb{R}^3 \mid x\ge 0, y\ge 0, z\ge 0, x^2+y^2+z^2\le 4\right\}$. Then $\iiint_E (x^2+y^2+z^2)dV =$____(5)____.

6. The volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x+2y+3z=9$ is ____(6)____.

7. Let $f(x,y)=x-y^2$. Find the tangent line to the level curve $f(x,y)=2$ at the point $(3,-1)$.
____(7)____

8. Find the volume of the solid above $z=\sqrt{x^2+y^2}$ and below $x^2+y^2+z^2=1$. ____(8)____

9. Find the area of the part of $x^2+y^2+z^2=100$ that lies within $x^2+y^2=10y$ and below the xy-plane. ____(9)____

10. Evaluate the integral $\iint_D e^{\frac{x+y}{x-y}}dA$, where D is the trapezoidal region with vertices (1,0), (2,0), (0,-2), and (0,-1). ____(10)____