九十五學年度第一學期微積分會考試題 甲卷

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。甲乙兩份試卷加答案卷共計 7 頁。

(3) 甲卷爲一般試卷，包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。乙卷爲挑戰題試卷，可自行決定是否作答，計 40 分，不佔學期成績。甲乙兩卷成績合計後，將做爲微積分獎給獎依據或教師加分參考。

(4) 乙卷採「延時加考」之方式進行，於測驗時間 110 分鐘結束，並回收甲卷後，再額外提供 30 分鐘時間作答乙卷。

(5) 請先確實填入相關個人資料。答題時請依題號空格作答，否則不予計分。

(6) 題目將於一月十六日於網站公佈。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. The total number of horizontal and vertical asymptotes of the graph of $f(x)=\dfrac{x^2}{(x-1)(x-3)}$ is

A) 1;

B) 2;

C) 3;

D) 4.

2. The value of $\lim_{x\to 1}(\dfrac{x}{x-1}-\dfrac{1}{\ln x})$ is

A) 0;

B) 1/2;

C) 1;

D) nonexistent.

3. The function $f(x)=x^{2/3}+|x^2-2|$ is differentiable for all real x except

A) 0;

B) 0 and $\sqrt{2}$;

C) $\pm\sqrt{2}$;

D) 0 and $\pm\sqrt{2}$.

4. If $f(x)=10^{\tan x}$, then $f'(\pi/4)$ equals

A) 10; B) 20; C) $2\ln 10$; D) $20\ln 10$.

5. Which of the following limits is wrong?

A) $\lim_{n\to\infty}(1+\frac{1}{2n+3})^{2n+3}=e$;

B) $\lim_{n\to\infty}(1+\frac{1}{n^2})^{n^2}=e$;

C) $\lim_{n\to\infty}(1+\frac{1}{n})^{2n}=e^2$;

D) $\lim_{n\to\infty}(1+\frac{1}{n^2})^{n}=\sqrt{e}$.

6. The length of the portion, cut off by the *x*-axis and *y*-axis, of any tangent line to the curve $x^{2/3}+y^{2/3}=4$ is

A) 2;
B) 4;
C) 8;
D) dependent on the tangent point.

7. $\int \tan^2 x dx =$

A) $\sec x + C$;

B) $\frac{1}{3}\tan^3 x + C$;

C) $\tan x - x + C$;

D) $\tan x + x + C$.

8. To find $\int \frac{dx}{\sqrt{(x^2-2x+10)^3}}$, we use the substitution

A) $\sin u = \frac{x-1}{3}$;

B) $\sin u = \frac{x-2}{3}$;

C) $\sin u = \frac{x-1}{9}$;

D) $\tan u = \frac{x-1}{3}$.

9. If *n* is a positive integer, then $\int_0^\infty x^n e^{-x} dx =$

A) 1; B) n; C) $n!$; D) ∞.

10. To find the surface area of the solid of revolution formed by rotating $y = x^2$ over $[0, 2]$ about the *x*-axis, we must evaluate

A) $\int_0^2 2\pi x^2 \sqrt{1+4x^2}\, dx$;

B) $\int_0^2 2\pi x^2 \sqrt{1+x^4}\, dx$;

C) $\int_0^2 2\pi x^2 \sqrt{1+2x}\, dx$;

D) $\int_0^2 4\pi x \sqrt{1+x^4}\, dx$.

◎ 複選擇題 (複選五題，每題五分，共二十五分，答錯不倒扣)

1. Which of the following statements are true?

A) If $f(x) > 0$ for all x and $\lim_{x\to 0} f(x)$ exists, then $\lim_{x\to 0} f(x) > 0$.

B) If f is concave upward on $[a,b]$, then f is increasing on $[a,b]$.

C) If $f'(x)$ exists and is nonzero for all real x, then f is a one-to-one function on $\mathbb{R}$.

D) If $f'(a)$ exists, then $\lim_{x\to a} f(x) = f(a)$.

2. Which of the following statements are true for $f(x) = (\ln x)/x$?

A) f is increasing on $(0,1)$.

B) f has the absolute maximum value $1/e$.

C) f is concave downward on $(0,1)$.

D) The graph of f has no inflection point.

3. How many real roots does the equation $x^4 + 4x + 1 = 0$ have?

A) at least 1;

B) at least 2;

C) at least 3;

D) at most 4.

4. Let f be a continuous function defined on $[0,1]$ and A be the limit

$$A = \lim_{x\to 0^+} x^2 \int_x^1 \frac{f(t)}{t^3} dt .$$

Which of the following statements are true?

A) $A = 0$;

B) $A = 1$;

C) $A = \dfrac{f(0)}{2}$;

D) $A = f(0)$.

5. It is well known that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. Which of the following statements are true?

A) $\int_{-\infty}^{\infty} e^{-x^2} dx = 0$;

B) $\int_{-\infty}^{\infty} x^2 e^{-x^2} dx = \frac{1}{2}\sqrt{\pi}$;

C) $\int_{-\infty}^{\infty} x^3 e^{-x^2} dx = 0$;

D) $\int_0^{\infty} x^{-a^2 x^2} dx = \sqrt{\pi},\ a > 0$.

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. $\lim_{x\to\infty}(\sqrt{9x^2 + x} - 3x) =$ ____(1)____.

2. If $f(x) = \sin(2x)$, then $f^{(103)}(x) =$ ____(2)____.

3. If $f(x) = 2x + \cos x$, then $(f^{-1})'(1) =$ ____(3)____

4. The area under the curve $y = \sin\sqrt{x}$ from $x = 0$ to $x = \pi^2$ is ____(4)____.

5. The perimeter of the curve $r = 1 - \cos\theta$ is ____(5)____.