

九十五學年度第二學期微積分會考試題 甲卷

說明:

- (1) 答題之前請先檢查所取得之試卷與答案卷是否正確。
- (2) 測驗時間 110 分鐘。甲乙兩份試卷加答案卷共計 6 頁。
- (3) 甲卷為一般試卷，包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。乙卷為挑戰題試卷，可自行決定是否作答，計 40 分，不佔學期成績。甲乙兩卷成績合計後，將做為微積分獎給獎依據或教師加分參考。
- (4) 乙卷採「延時加考」之方式進行，於測驗時間 110 分鐘結束，並回收甲卷後，再額外提供 30 分鐘時間作答乙卷。
- (5) 請先確實填入相關個人資料。答題時請依題號空格作答，否則不予計分。
- (6) 題目將於七月三日於網站公佈。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-2)^n (x-1)^n}{n}$ is

- A) $[1, 3)$ B) $[\frac{1}{2}, \frac{3}{2})$ C) $(\frac{1}{2}, \frac{3}{2}]$ D) $(\frac{1}{2}, \frac{3}{2})$.

2. If $f(x) = \cos(x^2)$, then $f^{(12)}(0) =$

- A) $\frac{1}{12!}$ B) $-\frac{1}{6!}$ C) $\frac{12!}{6!}$ D) $-\frac{12!}{6!}$.

3. Let L be the line passing through the point $(2, 1, -1)$ and parallel to the line $\frac{1}{3}x + 2 = y - 3 = -\frac{1}{2}z$. Which of the following is NOT the equation of L ?

A) $\begin{cases} x = 3t + 2 \\ y = t + 1 \\ z = -2t - 1 \end{cases}$

B) $\begin{cases} x - y + z = 0 \\ 2y + z - 1 = 0 \end{cases}$

C) $-2x + 2 = -6y + 4 = 3z + 1$

D) $x + y + 2z = 1$.

4. If θ is the angle between the planes $x - z = 1$ and $2x + y - 2z = 3$, then $\cos \theta$ equals

- A) $\frac{1}{\sqrt{2}}$ B) $\frac{4}{3\sqrt{2}}$ C) $\frac{2}{3\sqrt{2}}$ D) $\frac{1}{3\sqrt{2}}$.

5. The tangent plane to the surface $z = x^2 + 2y^2$ at the point $(1,1,3)$ is

- A) $2x - 2y = -z + 3$
- B) $2(x-1) + 4(y-1) = z - 3$
- C) $2x - 2y - z + 3 = 0$
- D) $2(x-1) + 4(y-1) + z - 3 = 0$.

6. If $z = f(x, y)$ with $x = r \cos \theta$ and $y = r \sin \theta$, and $\frac{\partial z}{\partial y} = \frac{\partial z}{\partial r} \sin \theta + \square \frac{\partial z}{\partial \theta} \cos \theta$, then $\square =$

- A) r
- B) r^2
- C) $\frac{1}{r}$
- D) $\frac{1}{r^2}$.

7. The value of $\iint_R \sin(x^2 + y^2) dA$, where $R = \{(x, y) | 1 \leq x^2 + y^2 \leq 4, y \geq 0\}$, is

- A) $\frac{\pi}{2}(\cos 1 - \cos 4)$
- B) $\frac{\pi}{2}(\sin 4 - \sin 1)$
- C) $\pi(\cos 1 - \cos 4)$
- D) $\pi(\sin 4 - \sin 1)$.

8. The value of $\iint_R e^{x+y} dA$, where $R = \{(x, y) | |x| + |y| \leq 1\}$, is

- A) $2e - \frac{2}{e}$
- B) $4e - \frac{4}{e}$
- C) $\frac{1}{2}e - \frac{1}{2e}$
- D) $e - \frac{1}{e}$.

9. The area of the part of the surface $z = x + 2y^2 + 3$ that lies above the triangle with vertices $(0,0)$, $(0,1)$ and $(2,1)$ is

- A) $\frac{17}{6}\sqrt{2}$
- B) $\frac{15}{6}\sqrt{2}$
- C) $\frac{13}{6}\sqrt{2}$
- D) $\frac{11}{6}\sqrt{2}$.

10. The value of $\iiint_E z dV$, which E is the tetrahedron bounded by the four planes

$x = 0, y = 0, z = 0$ and $x + y + z = 1$, is

- A) $\frac{1}{48}$
- B) $\frac{1}{24}$
- C) $\frac{1}{12}$
- D) $\frac{1}{6}$.

◎ 多選擇題 (多選五題, 每題五分, 共二十五分)

說明：每題的四個選項各自獨立，其中至少有一個選項是正確的。每題皆不倒扣，四個選項全部答對者得 5 分，只錯一個選項可得 3 分，錯兩個或兩個以上選項不給分。

1. Which of the following sequences are convergent as n approaches ∞ ?

- A) $n \cdot \sin \frac{1}{\sqrt{n}}$ B) $\arctan n$
- C) $(1 - \frac{2}{n})^n$ D) $\sqrt{n^2 + 1} - n$.

2. Which of the following series are convergent ?

- A) $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$ B) $\sum_{n=3}^{\infty} (-1)^n \tan \frac{\pi}{n}$
- C) $\sum_{n=1}^{\infty} \ln(\frac{n}{3n-1})$ D) $\sum_{n=1}^{\infty} \frac{\cos n}{n!}$.

3. Let

$$f(x, y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 1 & \text{if } (x, y) = (0, 0). \end{cases}$$

Which of the following statements are TRUE ?

- A) f is continuous at $(0, 0)$.
- B) $f_x(0, 0)$ exists.
- C) $f_y(0, 0)$ exists.
- D) f is differentiable at $(0, 0)$.

4. Let $f(x, y) = \ln(x^2 y)$ and $P = (1, 1)$.

Which of the following statements are TRUE ?

- A) The gradient of f at P is $(2, 1)$.
- B) The directional derivative of f at P in the direction of $(3, -4)$ is 2 .
- C) The maximum rate of change of f at P occurs in the direction of $(2, 1)$.
- D) The maximum rate of change of f at P is $\sqrt{5}$.

5. Let $f(x, y) = x^4 - 4xy + 2y^2 - 1$. Which of the following statements are TRUE ?

- A) f has 3 critical points.
- B) f has a local minimum at $(1, 1)$.
- C) f has an absolute minimum at $(-1, -1)$.
- D) f has a saddle point at $(0, 0)$.

◎ 填充題 (五題, 每題五分, 共二十五分, 答錯不倒扣)

1. If $\sum_{n=0}^{\infty} a_n x^n$ is the Maclaurin series of $1/(1+x)^2$, then $a_n =$ _____ (1).

2. The volume of the parallelepiped determined by the vectors $(1,0,-2)$, $(2,1,0)$ and $(3,1,1)$ is _____ (2).

3. If $z = f(x-2y)$, then $2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} =$ _____ (3)

4. The maximum value of $8x+10y$ subject to $x^2 + y^2 = 41$ is _____ (4).

5. The value of the iterated integral $\int_0^1 \int_x^1 \cos(y^2) dy dx$ is _____ (5).