九十六學年度第一學期微積分會考試題 甲卷

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。甲乙兩份試卷加答案卷共計 7 頁。

(3) 甲卷為一般試卷，包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。乙卷為挑戰題試卷，可自行決定是否作答，計 40 分，不佔學期成績。甲乙兩卷成績合計後，將做為微積分獎給獎依據或教師加分參考。

(4) 乙卷採「延時加考」之方式進行，於測驗時間 110 分鐘結束，並回收甲卷後，再額外提供 30 分鐘時間作答乙卷。

(5) 請先確實填入相關個人資料。答題時請依題號空格作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. When using the $\varepsilon-\delta$ definition to prove that

$$\lim_{x\to 1}\ln(3-2x)=0,$$

the largest δ for $\varepsilon=1$ is

(A) $e-1/2$; (B) $(e-1)/2$; (C) $1-e^{-1}/2$; (D) $(1-e^{-1})/2$.

2. The value of $\lim_{x\to 1}(\frac{x}{x-1}-\frac{1}{\ln x})$ is

(A) 0; (B) 1/2; (C) 1; (D) nonexistent.

3. If $f(x)=10^{\tan x}$, then $f'(\frac{\pi}{4})$ equals

(A) 10; (B) 20; (C) $2\ln 10$; (D) $20\ln 10$.

4. The value of $\lim_{x\to\infty}\frac{1}{x\ln x}\int_1^x \ln t\,dt$ is

(A) 1; (B) 2; (C) 0; (D) ∞.

5. Let $f(x)=2x+\cos x$. Find $(f^{-1})'(1)$.

(A) 0; (B) 1/2; (C) 1; (D) not exist.

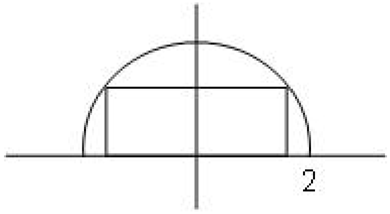
6. Let f be continuous on [0, 2] and differentiable in (0, 2). Suppose that $f(0)=2$ and $1<f'(x)<2$ for all x in (0, 2). Find a possible value of $f(2)$.

(A) 4; (B) 5; (C) 6; (D) 7.

7. Find the value of the integral $\int_0^1 x^3 e^{-x^2}\,dx$.

(A) $1+\frac{2}{e}$; (B) $\frac{1}{2}+\frac{1}{e}$; (C) $\frac{1}{2}-\frac{1}{e}$; (D) $1-\frac{2}{e}$.

8. A rectangle is inscribe in a semicircle of radius 2. What is the maximum area.



(A) $2\sqrt{2}$; (B) 4; (C) $4\sqrt{2}$; (D) 5.

9. The length of $r=e^{\frac{\theta}{2}}$, $0\le\theta\le 2$ is

(A) $\sqrt{2}e$; (B) $\sqrt{2}(e-1)$; (C) $\sqrt{5}e$; (D) $\sqrt{5}(e-1)$.

10. To find the surface area of the solid of revolution formed by rotating $y=x^2$ over $[0,2]$ about the x-axis, we must evaluate

(A) $\int_0^2 2\pi x^2\sqrt{1+4x^2}\,dx$;

(B) $\int_0^2 2\pi x^2\sqrt{1+x^4}\,dx$;

(C) $\int_0^2 2\pi x^2\sqrt{1+2x}\,dx$;

(D) $\int_0^2 4\pi x\sqrt{1+x^4}\,dx$.

◎ 複選擇題 (複選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

1. Which of the following functions are NOT continuous at $x=0$?

(A) $f_1(x)=\ln|x|$.

(B) $f_2(x)=\tan^{-1}(x)$.

(C) $f_3(x)=\begin{cases}\frac{\sin x}{x}, & \text{if } x\neq 0,\\ 0, & \text{if } x=0.\end{cases}$

(D) $f_4(x)=\begin{cases}\dfrac{x^2+2x}{x^2+x}, & \text{if } x\neq 0,\\ 2, & \text{if } x=0.\end{cases}$

2. Consider

$$f(x)=\begin{cases}x^2\sin(1/x), & \text{if } x\neq 0;\\ 0, & \text{if } x=0.\end{cases}$$

Which of the following statements are WRONG ?

(A) f is continuous on $\mathbb{R}$;

(B) f is differentiable on $\mathbb{R}$;

(C) $f'(0)=1$;

(D) f' is continuous at $x=0$.

3. Which of the following statements are TRUE for $f(x)=(\ln x)/x$?

(A) f is increasing on $(0,1)$;

(B) f has the absolute maximum value $1/e$;

(C) f is concave downward on $(0,1)$;

(D) The graph of f has no inflection point.

4, Which of the following statements are TRUE ?

(A) If f is continuous on $\mathbb{R}$, then $\dfrac{d}{dx}\int_0^x f(t)dt = f(x)$;

(B) If f is differentiable on $\mathbb{R}$, then $\int_0^x \dfrac{d}{dt} f(t)dt = f(x)$;

(C) If f is differentiable on $\mathbb{R}$, then f is continuous on $\mathbb{R}$;

(D) If f is differentiable at a, then its derivative f' is continuous at a.

5. Which of the following statements are TRUE for the ellipse $\dfrac{x^2}{4}+\dfrac{y^2}{9}=1$?

(A) The area of the ellipse is $2\int_{-2}^{2}\sqrt{4-x^2}\,dx$;

(B) The circumference of the ellipse is $\int_{-2}^{2}\sqrt{4+\dfrac{9x^2}{4-x^2}}\,dx$;

(C) The surface area of the solid obtained by the ellipse rotating about the x-axis is

$\int_{-2}^{2}\dfrac{3}{2}\pi\sqrt{16+5x^2}\,dx$;

(D) The volume of the solid obtained by the ellipse rotating about the x-axis is

$\int_{-2}^{2}\dfrac{3}{2}\pi(4-x^2)\,dx$.

◎ 填空題　(五題，每題五分，共二十五分，答錯不倒扣)

1. Suppose that $x^2+x\le f(x)\le x^3+1$ for all $x\in[0,2]$. Then $\lim_{x\to1} f(x)=$ ___(1)____.

2. Express $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{\pi}{n}\cdot\frac{\pi i}{n}\cdot\sin\frac{\pi i}{n}\right)$ as a definite integral.___(2)____

3. $\int_0^1 \frac{dx}{\sqrt{x^2+4}}=$ ____(3)____.

4. Evaluate $\int_{-\infty}^{\infty}\frac{dx}{x^2+2x+2}=$ ____(4)____.

5. The area under the curve $y=\sin\sqrt{x}$ from $x=0$ to $x=\pi^2$ is ___(5)____.