九十六學年度第二學期微積分會考試題 甲卷

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。甲乙兩份試卷加答案卷共計 8 頁。

(3) 甲卷為一般試卷，包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。乙卷為挑戰題試卷，可自行決定是否作答，計 40 分，不佔學期成績。甲乙兩卷成績合計後，將做為微積分獎給獎依據或教師加分參考。

(4) 乙卷採「延時加考」之方式進行，於測驗時間 110 分鐘結束，並回收甲卷後，再額外提供 30 分鐘時間作答乙卷。

(5) 請先確實填入相關個人資料。答題時請依題號空格作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. The interval of convergence of the series $\sum_{n=1}^{\infty}\frac{(-2)^n(x-1)^n}{n}$ is

(A) $[1,3)$; (B) $[\frac{1}{2},\frac{3}{2})$; (C) $(\frac{1}{2},\frac{3}{2}]$; (D) $(\frac{1}{2},\frac{3}{2})$.

2. For which α is the interval of convergence of

$$\sum_{n=1}^{\infty}\frac{n^{\alpha}}{n+1}x^n$$

equal to $[-1,1)$.

(A) 1; (B) 1/2; (C) -1/2; (D) -1.

3. If $\vec{F}(t)=\hat{i}+t\hat{j}+t^2\hat{k}$

$\vec{G}(t)=t\hat{i}+e^t\hat{j}+3\hat{k}$

Which of the following is not true

(A) $\vec{F}'(t)=\hat{j}+2t\hat{k}$;

(B) $G'(t)\times\vec{F}(t)=(t^2e^t)\hat{i}+t^2\hat{j}+(t-e^t)\hat{k}$;

(C) $(\vec{F}\times\vec{G})'(t)=(3-2te^t-t^2e^t)\hat{i}+(3t^2)\hat{j}+(e^t-2t)\hat{k}$;

(D) $(\vec{F}\times\vec{G})'(t)=\vec{F}'(t)\times\vec{G}(t)+\vec{F}(t)\times\vec{G}'(t)$.

4. $\displaystyle\lim_{(x,y)\to(0,0)}\frac{\sin(x^2+y^2)}{x^2+y^2}=$

(A) 1; (B) 0; (C) $\frac{1}{2}$; (D) does not exist.

5. Find the volume of the solid bounded by the planes $y=0, z=0$ and $z=1-x+y$ and the parabolic cylinder $y=1-x^2$

(A) $\frac{28}{15}$; (B) $\frac{23}{15}$; (C) $\frac{11}{6}$; (D) $\frac{13}{7}$.

6. The value of $\iint_R \sin(x^2+y^2)dA$, where $R=\{(x,y)\,|\,1\le x^2+y^2\le 4, y\ge 0\}$, is

(A) $\frac{\pi}{2}(\cos 1-\cos 4)$; (B) $\frac{\pi}{2}(\sin 4-\sin 1)$;

(C) $\pi(\cos 1-\cos 4)$; (D) $\pi(\sin 4-\sin 1)$.

7. Suppose f is continuous on $\mathbb{R}$. Then $\int_0^4\int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} f(x,y)dxdy=$

(A) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^4 f(r\cos\theta, r\sin\theta)rdrd\theta$;

(B) $\int_0^{\pi}\int_0^4 f(r\cos\theta, r\sin\theta)rdrd\theta$;

(C) $\int_0^{2\pi}\int_0^4 f(r\cos\theta, r\sin\theta)rdrd\theta$;

(D) None of above.

8. Which of the following is wrong.

(A) $\int_0^1\int_0^{1-z}\int_0^2 dxdydz=\int_0^1\int_0^{1-y}\int_0^2 dxdzdy$;

(B) $\int_0^1\int_0^{1-z}\int_0^2 dxdydz=\int_0^1\int_0^2\int_0^{1-x} dydxdz$;

(C) $\int_0^2\int_0^1\int_0^{1-z} dydzdx=\int_0^1\int_0^{1-y}\int_0^2 dxdzdy$;

(D) $\int_0^1\int_0^2\int_0^{1-y} dzdxdy=\int_0^2\int_0^1\int_0^{1-y} dzdydx$.

9. Find the surface area of the part of the surface $z=2xy$ that lies within the cylinder $x^2+y^2=4$.

(A) $\frac{\pi}{6}(17\sqrt{17}-1)$; (B) $\frac{\pi}{7}(19\sqrt{19}-1)$;

(C) $\frac{\pi}{8}(21\sqrt{21}-1)$; (D) $\frac{\pi}{9}(23\sqrt{23}-1)$.

10. The area of the part of the surface $z=x+2y^2+3$ that lies above the triangle with vertices $(0,0),(0,1)$ and $(2,1)$ is

(A) $\frac{17}{6}\sqrt{2}$; (B) $\frac{15}{6}\sqrt{2}$; (C) $\frac{13}{6}\sqrt{2}$; D) $\frac{11}{6}\sqrt{2}$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

1. Which of the following statements about the infinite sequence $\{a_n\}_{n=1}^{\infty}$ are true ?

(A) If $\lim_{n\to\infty} a_n = 0$, then $\lim_{n\to\infty}|a_n| = 0$;

(B) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty}\left(|a_n|+a_n\right)$ converges;

(C) If $\sum_{n=1}^{\infty}\left(|a_n|+a_n\right)$ converges, then $\sum_{n=1}^{\infty} a_n$ converges;

(D) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ converges.

2. Which of the following series are convergent ?

(A) $\sum_{n=1}^{\infty}\frac{n}{n^2+1}$; (B) $\sum_{n=3}^{\infty}(-1)^n\tan\frac{\pi}{n}$;

(C) $\sum_{n=1}^{\infty}\ln(\frac{n}{3n-1})$; (D) $\sum_{n=1}^{\infty}\frac{\cos n}{n!}$.

3. Which of the following statements about a 2 variables function $f(x,y)$ are true ?

(A) If the partial derivatives f_x and f_y exist at (x_0,y_0), then $f(x,y)$ is continuous at (x_0,y_0);

(B) If f_x and f_y exist near (x_0,y_0) and are continuous at (x_0,y_0), then $f(x,y)$ is differentiable at (x_0, y_0);

(C) If $f(x,y)$ is differentiable at (x_0, y_0), then f is continuous at (x_0, y_0);

(D) If $f(x,y)$ is defined on a disk D that contains the point (x_0, y_0) and the functions f_{xy}, f_{yx} are both continuous on D, then $f_{xy}(x_0, y_0) = f_{yx}(x_0, y_0)$.

4, Let

$$f(x,y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2} & \text{if } (x,y) \neq (0,0), \\ 1 & \text{if } (x,y) = (0,0). \end{cases}$$

Which of the following statements are TRUE ?

(A) f is continuous at $(0,0)$;

(B) $f_x(0,0)$ exists;

(C) $f_y(0,0)$ exists;

(D) f is differentiable at $(0,0)$.

5. Consider the following function

$$f(x,\ y) = x^3 + 6xy - 3y^2 + 2.$$

Which of the following statements are TRUE ?

(A) f has 3 critical points;

(B) f has a local maximum at $(-2,-2)$;

(C) f has a local minimum at $(-2,-2)$;

(D) f has a saddle point at (0, 0).

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. Find the limit $\lim_{t\to\infty}(\tan^{-1} t,\ e^{-4t}, \dfrac{\ln t}{t}) =$ ___(1)____.

2. Find the first four terms ___(2)____ of the Maclaurin series of

$$f(x) = \frac{\cos x}{1-x}.$$

3. Let

$$w = ze^{\frac{x}{y}}, \quad x = \sin(t^2 s), \quad y = (1+st)(1+t^2)^{-\frac{1}{2}}, \quad z = t(t^2 + s^2)^{-\frac{3}{2}}.$$

Then, $\partial w / \partial s \,|_{(s,t)=(0,1)} =$ _____(3)_____.

4. ____(4)____ is the highest point on the plane curve that is given by the intersection of $x + y + z = 0, x^2 + y^2 + z^2 = 1$ ..

5. $\int_0^4 \int_{\sqrt{x}}^2 \sqrt{1 + y^3}\, dydx =$ ___(5)____.