九十七學年度第一學期微積分會考試題 甲卷

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。甲乙兩份試卷加答案卷共計 7 頁。

(3) 甲卷為一般試卷，包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。乙卷為挑戰題試卷，可自行決定是否作答，計 40 分，不佔學期成績。甲乙兩卷成績合計後，將做為微積分獎給獎依據或教師加分參考。

(4) 乙卷採「延時加考」之方式進行，於測驗時間 110 分鐘結束，並回收甲卷後，再額外提供 30 分鐘時間作答乙卷。

(5) 請先確實填入相關個人資料。答題時請依題號空格作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. When using the ε-δ definition to prove that $\lim_{x\to 0}(x^2-2x)=0$ the largest δ for $\varepsilon=1$ is

(A) $2+\sqrt{6}/2$; (B) $1+\sqrt{6}/2$; (C) $\sqrt{6}/2$; (D) $\sqrt{6}/2-1$.

2. Which of the following statements is true?

(A) $\lim_{x\to\infty}\frac{\sin x}{x}=1$; (B) $\lim_{x\to 0^+} x\sin\frac{1}{x^2}$ does not exist;

(C) $\lim_{x\to 0^+}\frac{\sin^{-1}x}{x}=1$; (D) $\lim_{x\to\frac{1}{2}}\frac{\sin^{-1}x}{x}=\frac{2}{\pi}$.

3. What is the value of k that makes the function

$$f(x)=\begin{cases}\dfrac{9x-3\sin 3x}{5x^3}, & x\neq 0,\\ k, & x=0,\end{cases}$$

continuous at $x=0$?

(A) $\frac{21}{10}$; (B) $\frac{11}{5}$; (C) $\frac{27}{10}$; (D) 2.

4. The value of $\lim_{x\to 0}\left(\frac{x+1}{x}-\frac{1}{\ln(x+1)}\right)$ is

(A) 0; (B) $\frac{1}{2}$; (C) 1; (D) nonexistent.

5. Let f be continuous on $[0,2]$ and differentiable in $(0,2)$. Suppose that $f(0)=1$ and $2<f'(x)<3, \forall x\in(0,2)$. Find a possible value of $f(2)=$

(A) 5; (B) 6; (C) 7; (D) 8.

6. Evaluate $\int_0^2 x^3 e^{-x^2}dx$

(A) $1+\frac{2}{e^4}$; (B) $\frac{1}{2}-\frac{5}{2}\cdot\frac{1}{e^4}$;

(C) $\frac{1}{2}-\frac{2}{e^4}$; (D) $1-\frac{4}{e^4}$.

7. Evaluate $\int_0^{1/3}\frac{dx}{\sqrt{1+9x^2}}$

(A) $\frac{1}{3}\ln(\sqrt{3}+1)$; (B) $\frac{1}{3}\ln(\sqrt{2}+1)$;

(C) $\frac{1}{\sqrt{3}}\ln(\sqrt{3}+1)$; (D) $\ln(\sqrt{2}+1)$.

8. The volume generated by revoluting the region bounded by $y=x^2$ and $y=4x-x^2$ about the line $x=4$ is

(A) 8π; (B) 16π; (C) 32π; (D) 4π.

9. To find the surface area of the solid of revolution formed by rotating $y=x^3$ over $[0,1]$ about the *x*-axis, we must evaluate

(A) $\int_0^1 2\pi x^3\sqrt{1+x^3}dx$; (B) $\int_0^1 2\pi x\sqrt{1+9x^4}dx$;

(C) $\int_0^1 2\pi x^3\sqrt{1+9x^4}dx$; (D) $\int_0^1 2\pi x^2\sqrt{1+x^6}dx$.

10. The area between the circle $r=5\cos\theta$ and the limacon $r=2+\cos\theta$ is

(A) $\frac{5}{6}\pi-\sqrt{3}$; (B) $\frac{25}{12}\pi-\frac{25}{8}\sqrt{3}$; (C) $\frac{3}{2}\pi-\frac{25}{8}\sqrt{3}$; D) $\frac{43}{12}\pi-\sqrt{3}$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

1. Which of the following statements are true for $f(x)=(\ln x^2)/x$?
   (A) f is increasing on $(0,1)$;
   (B) f has the absolute maximum value $\frac{1}{e}$;
   (C) f is concave downward on $(0,1)$;
   (D) The graph of f has no inflection point.

2. Let $f:[0,1]\to\mathbb{R}$, $a\in(0,1)$. Which of the following statements are false?
   (A) If $f(x)$ has the maximum at $x=a$, then $f'(a)=0$;
   (B) If $f''(a)=0$, then a is an inflection point;
   (C) If f is continuous on $[0,1]$, then $\{f(x)\in\mathbb{R}\mid x\in[0,1]\}$ is a closed interval;
   (D) If f is convex upward on $[0,1]$, then f is increasing on $[0,1]$.

3. Let $I=\lim_{n\to\infty}\sum_{i=1}^{n}\left\{\frac{2}{n}\left[4-3\left(\frac{2i}{n}\right)^2+6\left(\frac{2i}{n}\right)^5\right]\right\}$ which of the following statements are true?
   (A) $I=4$;
   (B) $I=64$;
   (C) $I=\int_0^1(4-6x^2+12x^5)dx$;
   (D) $I=\int_0^2(4-3x^2+6x^5)dx$.

4. Which of the following statement is true for the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$?
   (A) The area of the ellipse is $3\int_{-2}^{2}\sqrt{4-x^2}dx$;
   (B) The circumference of the ellipse is $\int_{-2}^{2}\sqrt{4+\frac{9x^2}{4-x^2}}\,dx$;
   (C) The surface area of the solid obtained by the ellipse rotating about the x-axis is $\int_{-2}^{2}\frac{3}{2}\pi\sqrt{9+5x^2}dx$;
   (D) The volume of the solid obtained by the ellipse rotating about the x-axis is $\int_{-2}^{2}\frac{3}{2}\pi(4-x^2)dx$.

5. For the parametric equation

$$x = 2 + \sec\theta,$$
$$y = 1 + 2\tan\theta.$$

What are the values $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$?

(A) 4; (B) $\frac{3}{\sqrt{2}}$; (C) $-6\sqrt{3}$; (D) $3\sqrt{2}$.

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. Let $f(u) = \cot(\pi u/10)$ and $u = g(x) = 5\sqrt{x}$. Then, $(f \circ g)'(1)$ = ___(1)___.

2. Suppose we know that $f(x) = 16x^5 + 7x + 1$ has an inverse function f^{-1}. Then $(f^{-1})'(5)$ = ___(2)___.

3. Let $h(x) = \int_0^{x^2} \sqrt{1+s^3}\,ds$ then $h'(x)$ is ___(3)___.

4. $\int \tan^{-1} x\,dx$ = ___(4)___.

5. The length of the logarithmic spiral $r = e^{3\theta}$, $0 \le \theta \le 2$ is ___(5)___.