九十七學年度第二學期微積分會考試題 甲卷

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。甲乙兩份試卷加答案卷共計 9 頁。

(3) 甲卷爲一般試卷，包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。乙卷爲挑戰題試卷，可自行決定是否作答，計 40 分，不佔學期成績。甲乙兩卷成績合計後，將做爲微積分獎給獎依據或教師加分參考。

(4) 乙卷採「延時加考」之方式進行，於測驗時間 110 分鐘結束，並回收甲卷後，再額外提供 30 分鐘時間作答乙卷。

(5) 請先確實填入相關個人資料。答題時請依題號空格作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. The interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n (x+1)^n}{2^n n}$ is

(A) $(-3/2, -1/2]$, (B) $[-3/2, -1/2)$,

(C) $(-3, 1]$, (D) $[-3, 1)$.

2. The set of p for which the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^p}$ converges is

(A) $(-\infty, \infty)$, (B) $(-\infty, 1)$,

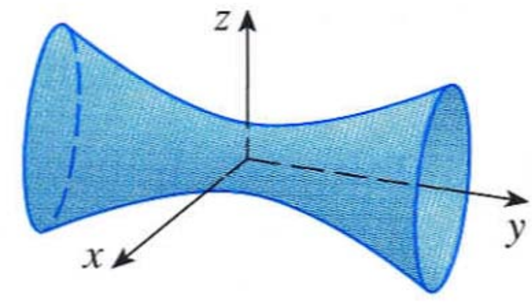
(C) $(1, \infty)$, (D) $[1, \infty)$.

3. Assume that $f(x) = \int_2^x \frac{e^t - 1}{t} dt$. Then $f^{(6)}(0) = ?$

(A) $\frac{1}{6!}$, (B) 1,

(C) $\frac{1}{6 \cdot (6!)}$, (D) $\frac{1}{6}$.

4. The graph



is the equation of

(A) $x^2 + 4y^2 + 9z^2 = 1$, (B) $x^2 - y^2 + z^2 = 1$,

(C) $-x^2 + y^2 - z^2 = 1$, (D) $y^2 = x^2 + 2z^2$.

5. $\lim_{(x,y)\to(0,0)} \dfrac{\sin(x^2 - y^2)}{x^2 + y^2} =$

(A) 1, (B) 0, (C) $\dfrac{1}{2}$, (D) does not exist.

6. Find $\dfrac{\partial z}{\partial y}\Big|_{(x,y)=(0,0)}$ of $\sin(xyz) = x + y - z$.

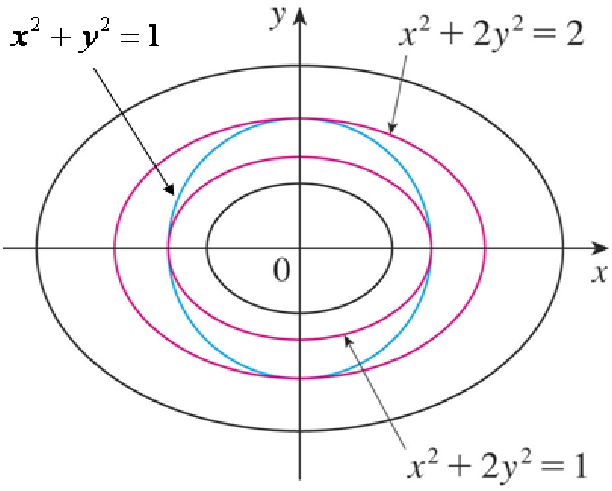
(A) 0, (B) 1, (C) 2, (D) 1/2.

7. The value of $\iint_R \sin(x^2 + y^2)\,dA$, where $R = \{(x, y) : 4 \le x^2 + y^2 \le 9, x \ge 0, y \ge 0\}$, is

(A) $\dfrac{\pi}{4}(\cos 9 - \cos 4)$ (B) $\dfrac{\pi}{2}(\cos 4 - \cos 9)$

(C) $\dfrac{\pi}{2}(\cos 9 - \cos 4)$ (D) $\dfrac{\pi}{4}(\cos 4 - \cos 9)$

8. Find the extreme values of $f(x,y) = x^2 + 2y^2$ on the disk $x^2 + y^2 \le 1$. Pictures below are the level curves of $f(x,y)$ and a circle with equation $x^2 + y^2 = 1$.



(A) minimum=0 and maximum=1,
(B) minimum=0 and maximum=2,
(C) minimum=1 and maximum=2,
(D) minimum=1 and maximum=1.

9. Evaluate $\iint_D (x+y)^2 dxdy$, where D is the parallelogram bounded by the lines $x+y=0$, $x+y=1$, $2x-y=0$, and $2x-y=3$.

(A) 1, (B) 1/2, (C) 2, (D) 1/3.

10. Which of the following triple integrals is different from the others ?

(A) $\int_0^1 \int_0^x \int_0^y f(x,y,z)\,dz\,dy\,dx$,

(B) $\int_0^1 \int_z^1 \int_y^1 f(x,y,z)\,dx\,dy\,dz$,

(C) $\int_0^1 \int_0^y \int_y^z f(x,y,z)\,dx\,dz\,dy$,

(D) $\int_0^1 \int_0^x \int_x^z f(x,y,z)\,dy\,dz\,dx$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

1. Which of the following series are convergent ?

(A) $\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{n^2}\right)$,

(B) $\sum_{n=3}^{\infty} (-1)^n \tan\frac{\pi}{n}$,

(C) $\sum_{n=1}^{\infty} \frac{n}{100n^2+1}$,

(D) $\sum_{n=1}^{\infty} \frac{\sin n}{n!}$.

2. Which of the following statements is true ?

(A) Assume that the series $\sum_{n=1}^{\infty} a_n$ converges. Then the series $\sum_{n=1}^{\infty} (-1)^n a_n$ also converges.

(B) Assume that the series $\sum_{n=1}^{\infty} a_n$ converges. Then the series $\sum_{n=1}^{\infty} a_n^2$ also converges.

(C) Assume that the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge. Then the series $\sum_{n=1}^{\infty} (a_n + b_n)$ also converges.

(D) Assume that the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ diverge. Then the series $\sum_{n=1}^{\infty} (a_n + b_n)$ also diverges.

3. Let $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 - 4$. Which of the following is true ?

(A) (0,0) is a local maximum,

(B) (2,0) is a local maximum,

(C) (1,1) and (1,-1) are saddle points.

(D) (2,0) and (0,0) are saddle points.

4. Let $f(x,y)=\begin{cases}\frac{y^2-x^2}{x^2+y^2}, & \text{if } (x,y)\neq(0,0),\\ 1, & \text{if } (x,y)=(0,0).\end{cases}$

Which of the following statements are TRUE ?

(A) f is continuous at (0,0),

(B) $f_x(0,0)$ exists,

(C) $f_y(0,0)$ exists,

(D) f is differentiable at $(x,y)\neq(0,0)$.

5. Which of the following statements is true ?

(A) If $f(x,y)$ has two local maxima, then f muse have a local minimum.

(B) If the partial derivatives f_x and f_y both exist near (a,b) and are continuous at (a,b), then f is differentiable at (a,b).

(C) If the mixed partial derivatives f_{xy} and f_{yx} both exist at (a,b), then $f_{xy}(a,b)=f_{yx}(a,b)$.

(D) If f has a local maximum and is differentiable at (a,b), then $\nabla f(a,b)=0$.

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. Let $f(x)=\dfrac{x}{\sqrt{1+x^4}}$ and $f(x)=\sum_{n=0}^{\infty}a_n x^n$ be its Maclaurin series. Then $(a_1,a_2,\cdots,a_6)=$ ____(1)____ .

2. The arc length of the curve $(\cos t,\sin t,t)$ from $t=0$ to $t=4\pi$ is ____(2)____ .

3. Find the directional derivative of the function $f(x,y,z)=\dfrac{x}{y+z}$ at the point (4,1,1) in the direction of the vector $\vec{u}=\langle 1,1,1\rangle$. ____(3)____

4. Evaluate $\int_0^4\int_{\sqrt{y}}^2 \dfrac{1}{x^3+1}\,dxdy$. ____(4)____

5. Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$ (see the figure below.)

(5)