九十八學年度第一學期微積分會考試題 (A 卷)

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。試卷加答案卷共計 5 頁。

(3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4) 請先確實在答案卷填入相關個人資料。答題時請依題號空格作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. When using the ε-δ definition to prove that $\lim_{x\to 0}(2x-x^2)=0$ for $\varepsilon=1$ the largest δ among the below answers is

(A) $1+\sqrt{6}/2$; (B) $\sqrt{6}/2$; (C) $\sqrt{6}/2-1$; (D) $2+\sqrt{6}/2$.

2. Let $F(x^3)=f(g(x^4))$, then which of following is true?

(A) $F'(x)=4x^3f'(g(x^4))g'(x^4)$; (B) $F'(x)=\dfrac{4x}{3}f'(g(x^4))g'(x^4)$;

(C) $F'(x)=\dfrac{4}{3}x^{1/3}f'(g(x^{4/3}))g'(x^{4/3})$; (D) $F'(x)=f'(g(x^4))g'(x^4)$.

3. Suppose f is continuous on $(0,\infty)$ and, for $x>0$, $\int_0^{x^2} f(t)dt=1+x$. Then $f(4)=$

(A) $\dfrac{1}{4}$; (B) $\dfrac{1}{2}$; (C) 1; (D) none of the above.

4. The value of $\lim_{x\to 0}\left(\dfrac{1}{\ln(x+1)}-\dfrac{x+1}{x}\right)$ is

(A) 0; (B) $-\dfrac{1}{2}$; (C) -1; (D) nonexistent.

5. Evaluate the limit $\lim_{n\to\infty} \frac{1}{2n}\sum_{i=1}^{2n} \frac{1}{1+\left(\frac{2i}{n}\right)^2}$ as a definite integral.

(A) $\int_0^1 \frac{1}{1+2x^2}dx$; (B) $\int_0^2 \frac{1}{2(1+2x^2)}dx$;

(C) $\int_0^1 \frac{1}{2(1+2x^2)}dx$; (D) $\int_0^2 \frac{1}{2(1+2x)^2}dx$.

6. Evaluate the integral $\int_0^1 x^5 e^{-x^3} dx$

(A) $\frac{1}{2}$; (B) $\frac{1}{3}$;

(C) $1-\frac{1}{3}e^{-1}$; (D) $\frac{1}{3}-\frac{2}{3}e^{-1}$.

7. For which positive real number b the average value of $f(x)=2+6x-3x^2$ on the interval $[0,b]$ is largest?

(A) $b=\frac{3-\sqrt{5}}{2}$; (B) $b=1.5$;

(C) $b=\frac{3+\sqrt{5}}{2}$; (D) $b=0.000001$.

8. Suppose $f(\theta)>0$ for $\theta\in[a,b]$ and $f(\theta)$ is continuous on $[a,b]$. Which of the following statement is always true about the arc length L of the polar curve $r=f(\theta)$ for $\theta\in[a,b]$?

(A) $L<\int_a^b f(\theta)d\theta$; (B) $L=\int_a^b f(\theta)d\theta$;

(C) $L>\int_a^b f(\theta)d\theta$; (D) None of the above is always true.

9. To find the surface area of the solid of revolution formed by rotating $y=x^3$ over $[0,2]$ about the *x*-axis, we must evaluate

(A) $\int_0^2 2\pi x\sqrt{1+9x^4}dx$; (B) $\int_0^2 2\pi x^3\sqrt{1+9x^4}dx$;

(C) $\int_0^2 2\pi x^2\sqrt{1+x^6}dx$; (D) $\int_0^2 2\pi x^3\sqrt{1+x^3}dx$.

10. A plane flying horizontal at an altitude of 1 mile and a speed of 500 miles/hour passes directly over a radar station. Which of the following statement about the rate r of change of the distance from the plane to the station when it is 3 miles away from the station is true.

(A) $1 < r < 500$; (B) $r = 500$;

(C) $r > 500$; (D) $r = \frac{2\sqrt{2}}{3}$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

1. Which of the following statements are true for $f(x) = (\ln x^2)/x$?

(A) f has the absolute maximum value $\frac{1}{e}$;

(B) f is concave downward on $(0,1)$;

(C) The graph of f has no inflection point;

(D) f is increasing on $(0,1)$.

2. Which of the following statements about the real function $f(x)$ on (a,b) are true?

(A) If $f(x)$ has a local maximum at some point $c \in (a,b)$ then $f'(c)$ does not exist or $f'(c) = 0$.

(B) If $f(x)$ is continuous on (a,b) then $f(x)$ must have an absolute maximum on (a,b).

(C) If $f^2(x)$ is differentiable on (a,b), then $f(x)$ must be differentiable on (a,b).

(D) If $f''(x)$ exists on (a,b), then $f'(x)$ is continuous on (a,b).

3. Which of the following are true?

(A) $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$; (B) $\lim_{x \to \infty} \frac{\sin(x)}{x} = 0$;

(C) $\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$ does not exist; (D) $\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right) = 1$.

4. Let $y = 6x^3 + 5x - 3$ represent a plane curve. Which of the following can not be the slope of a tangent line of the curve?

(A) 4; (B) 5; (C) 10; (D) 1.

5. Let $f(x)$ be a real function such that $f'''(x)$ exists for all real numbers x. Which of the following are the right expressions of $f''(x)$?

(A) $f''(x) = \lim_{h \to 0} \frac{f(x+h) + f(x-h) - 2f(x)}{h^2}$;

(B) $f''(x) = \lim_{h \to 0} \frac{f(x+h) + f(x-h) - 2f(x)}{2h}$;

(C) $f''(x) = \lim_{h \to 0} \frac{f'(x+h) + f'(x-h) - 2f'(x)}{2h}$;

(D) $f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x-h)}{2h}$.

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. $y =$ ____(1)____ is the equation of the tangent line to the curve $y = 2 - x^3$ at the point $(1,1)$.

2. $\lim_{x \to \infty} \left(1 - \frac{3}{x} + \frac{5}{x^2}\right)^x =$ ____(2)____.

3. If $h(2) = 4$ and $h'(2) = -2$, find $\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2} =$ ____(3)____.

4. $\int_0^1 \frac{2x}{1+x^4} dx =$ ____(4)____.

5. Find the value $m =$ ____(5)____ such that the line $y = mx$ and the curve $y = x/(x^2+1)$ enclose a region of area $-0.8 + \ln 5$.