九十八學年度第二學期微積分會考試題 (A 卷)

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。試卷加答案卷共計 6 頁。

(3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. What is the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n (x^2+3)^n}{n6^{n+1}}$?

   (A) $\left(-\sqrt{3}, \sqrt{3}\right)$, (B) $\left[-\sqrt{3}, \sqrt{3}\right]$,

   (C) $\left(3-\sqrt{6}, 3+\sqrt{6}\right)$, (D) $\left[3-\sqrt{6}, 3+\sqrt{6}\right]$.

2. $\lim_{(x,y)\to(0,0)} \frac{\sin(2x^2+y^2)}{2x^2+y^2} =$

   (A) 1, (B) 0, (C) $\frac{1}{2}$, (D) does not exist.

3. The coefficient of x^3 in the Maclaurin series of $f(x)=(x-1)e^{x-1}$ is

   (A) $1/2$, (B) 3, (C) $2/e$, (D) $1/(3e)$.

4. The value of $\iint_R \sin(x^2+y^2)\,dA$, where $R=\left\{(x,y): 1\le x^2+y^2\le 4, x\ge 0, y\ge 0\right\}$, is

   (A) $\frac{\pi}{2}(\cos 1-\cos 4)$, (B) $\frac{\pi}{4}(\cos 1-\cos 4)$,

   (C) $\frac{\pi}{2}(\cos 4-\cos 1)$, (D) $\frac{\pi}{4}(\cos 4-\cos 1)$.

5. The limit of $\lim_{x\to 0} \frac{\cos(2x)-1+2x^2}{x(\sin x - x)}$ is equal to

   (A) 2, (B) 0, (C) -2, (D) -4.

6. Evaluate the double integral $\iint_D xy\,dA$, where D is the region in the first quadrant bounded by the four curves $x^2+y^2=4$, $x^2+y^2=9$, $x^2-y^2=1$, $x^2-y^2=4$.

(A) $9/8$, (B) $11/8$, (C) $13/8$, (D) $15/8$.

7. Let C be the curve determined by the vector function $r(t)=\left\langle \sqrt{2}\,t, e^t, e^{-t}\right\rangle$. The length of C between $r(0)$ and $r(\ln 2)$ is

(A) 1, (B) $3/2$, (C) 2, (D) $\ln 2$.

8. Find $\left.\frac{\partial z}{\partial x}\right|_{(x,y)=(0,0)}$ of $\arctan(2x-y+2z)=3xyz$.

(A) -1, (B) 0, (C) 1, (D) 2.

9. The area of the triangle with vertices $(1,4,6)$, $(-3,0,2)$ and $(0,1,5)$ is

(A) $\sqrt{2}$, (B) $2\sqrt{2}$, (C) $4\sqrt{2}$, (D) $8\sqrt{2}$.

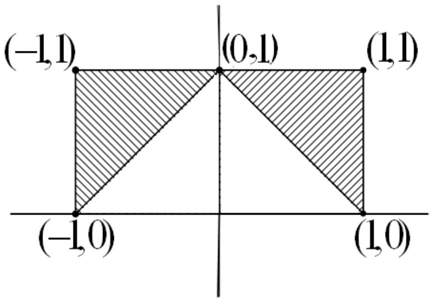
10. Evaluate the double integral $\iint_D x^2\,dA$, where D is the union of gray regions in the following figure.

(A) 0,

(B) $1/2$,

(C) 1,

(D) 2.



◎ 多選擇題 (多選 五題，每題五分，

共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

1. Let a_n, b_n be sequences of real numbers. Which of the following statements must be true？

(A) If $\sum_n |a_n|$ converges, then $\sum_n a_n$ converges.

(B) If $\sum_n a_n$ converges, then $\sum_n (-1)^n |a_n|$ converges.

(C) If $\sum_n a_n$ and $\sum_n b_n$ are convergent, then $\sum_n (a_n+b_n)$ is convergent.

(D) If $\sum_n a_n$ and $\sum_n b_n$ are convergent, then $\sum_n (a_n b_n)$ is convergent.

2. Let $f(x,y)=\ln(x^2y)$ and $P=(1,1)$. Which of the following statements are TRUE？

(A) The gradient of f at P is $(2,1)$.

(B) The directional derivative of f at P in the direction of $(3,-4)$ is 2.

(C) The maximum rate of change of f at P occurs in the direction of $(2,1)$.

(D) The maximum rate of change of f at P is $\sqrt{5}$.

3. Which of the following series are convergent？

(A) $\sum_{n=1}^{\infty} \ln\left(\frac{n+\ln n}{n}\right)$, (B) $\sum_{n=2}^{\infty} \frac{1}{n\log n}$,

(C) $\sum_{n=1}^{\infty} (-1)^n \sin\frac{\pi}{n}$, (D) $\sum_{n=1}^{\infty} \frac{e^n}{n!}$.

4. Let $f(x,y)=\begin{cases} \frac{x^2-y^2}{x^2+y^2}, & \text{if } (x,y)\neq(0,0), \\ 1, & \text{if } (x,y)=(0,0). \end{cases}$

Which of the followings statements are TRUE？

(A) f is continuous at $(0,0)$,

(B) f is differentiable at $(x,0)$ for $x\neq 0$,

(C) $f_x(0,0)$ exists,

(D) $f_y(0,0)$ exists.

5. Let a_n be real numbers and $f(x)=\sum_{n=0}^{\infty} a_n(x-1)^n$. Which of the following must be true？

(A) If $f(x_0)$ is convergent, then $f(y_0)$ is convergent for $|y_0|<|x_0|$.

(B) If $f(x_0)$ is divergent, then $f(y_0)$ is divergent for $|y_0|>|x_0|$.

(C) If $f(|x_0|)$ is convergent, then $f(y_0)$ is convergent for $0<y_0<|x_0|$.

(D) If $f(x_0)$ is divergent, for some $x_0<0$, then $f(y_0)$ is divergent for $y_0<x_0$.

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. Find the tangent plane to the surface $\sin(x+y+z)=xyz$ at the point $(1,-1,0)$.
____(1)____

2. The set of p such that the series $\sum_{n=1}^{\infty}\ln(1+1/n^p)$ converges is ____(2)____.

3. Find the extreme values of $f(x,y,s,t)=x+y+s+t$ subject to the constraint $x^2+y^2+s^2+t^2=1$. ____(3)____

4. The 3rd-degree Taylor polynomial of $f(x)=\sin(e^{x^2}-1)$ about 0 is ____(4)____.

5. Evaluate the integral $\int_0^4\int_0^1\int_{2y}^2 \frac{4\cos x^2}{2\sqrt{z}}\,dxdydz$. ____(5)____.