

# 九十九學年度第一學期微積分會考試題 (A 卷)

說明:

- (1) 答題之前請先檢查所取得之試卷與答案卷是否正確。
- (2) 測驗時間 110 分鐘。試卷加答案卷共計 6 頁。
- (3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。
- (4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

## ◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. How many points of intersection are there between  $y = x^2$  and  $y = 2^x$  ?  
(A) 0; (B) 1; (C) 2; (D) 3.

2. Consider  $f(x) = \begin{cases} -5x+2 & \text{if } x \geq 0, \\ \frac{-(x+1)^2}{4} & \text{if } x < 0. \end{cases}$

When using the  $\varepsilon - \delta$  definition to prove that  $\lim_{x \rightarrow -1} f(x) = 0$ , the largest  $\delta$  for  $\varepsilon = 1$  is

- (A) 2; (B) 1; (C)  $1\frac{2}{5}$ ; (D)  $1\frac{3}{5}$ .

3. Evaluate the limit of  $\lim_{x \rightarrow 0} \left( \frac{1}{\ln(x+1)} - \frac{x+1}{x} \right)^2$ .

- (A) 0; (B)  $\frac{1}{4}$ ; (C) 1; (D) nonexistent.

4. Evaluate the limit  $\lim_{n \rightarrow \infty} \frac{1}{2n} \sum_{i=1}^n \left( \frac{e^{\frac{4+6i}{n}}}{3 + \frac{3i}{n}} \right)$  as a definite integral.

- (A)  $\int_2^5 \frac{e^{2x}}{6(1+x)} dx$ ; (B)  $\int_0^3 \frac{e^{4+2x}}{2(3+x)} dx$ ;  
(C)  $\int_0^1 \frac{e^{4+x^6}}{2(3+x^3)} dx$ ; (D)  $\int_3^6 \frac{e^{1+2x}}{6x} dx$ .

5. Which one of the following statements is TRUE for  $\int_0^1 e^{-x^2} dx = L$  ?
- (A)  $\frac{1}{2}(1+e^{-1}) < L < 1$ ; (B)  $L = 1$ ; (C)  $\frac{1}{2}(e^{-1} + e^{-4}) < L < 1$ ; (D)  $L > 1$ .
6. Evaluate the integral  $\int_0^1 \frac{x^2 + x + 2}{x^2 + 1} dx$ .
- (A)  $2 - 2 \ln 2$ ; (B)  $\frac{4 \ln 2 - \pi}{6}$ ; (C)  $\frac{\pi + 4 + 2 \ln 2}{4}$ ; (D)  $\frac{\pi + 8 - \ln 2}{8}$ .
7. Find the surface area of the solid of revolution formed by rotating  $y = x^4$  over  $[0, 3]$  about the  $x$ -axis.
- (A)  $\int_0^3 2\pi x \sqrt{1 + 16x^6} dx$ ; (B)  $\int_0^3 2\pi x \sqrt{1 + x^3} dx$ ;  
 (C)  $\int_0^3 2\pi x^4 \sqrt{1 + 16x^6} dx$ ; (D)  $\int_0^3 2\pi x^4 \sqrt{1 + x^3} dx$ .
8. Evaluate the integral  $\int_e^\infty \frac{1 + e^{-x^2}}{x} dx$ .
- (A)  $\frac{1}{e}$ ; (B) 1; (C)  $e^{-e^2}$ ; (D) divergent.
9. Find the area of the region bounded by the  $x$ -axis,  $x = \pi r$  and the half arch of the cycloid  $x = r(\theta - \sin \theta)$ ,  $y = r(1 - \cos \theta)$  with  $r > 0$  and  $0 \leq \theta \leq \pi$ .
- (A)  $\pi r^2$ ; (B)  $\frac{3}{2} \pi r^2$ ; (C)  $3\pi r^2$ ; (D)  $3\pi r^3$ .
10. Find the area outside the circle  $r = 3$  and inside the polar curve  $r = 2 + 2 \cos \theta$ .
- (A)  $\pi$ ; (B)  $4 + \frac{\sqrt{3}}{2} - \frac{\pi}{2}$ ;  
 (C)  $\frac{9}{2} \sqrt{3} - \pi$ ; (D)  $2 + 7\sqrt{2} - \pi$ .

◎ 多選擇題 (多選五題, 每題五分, 共二十五分。答錯一個選項扣兩分, 錯兩個選項以上不給分, 分數不倒扣)

1. Which of the following statements are TRUE for  $f(x) = \frac{\ln x^2}{2x^2}$ ?

- (A)  $f$  is increasing on  $(-1, 0)$ .
- (B)  $f$  is concave downward on  $(-1, 0)$ .
- (C) The graph of  $f$  has only one inflection point.
- (D)  $f$  has the absolute maximum value  $\frac{1}{2e}$ .

2. Which of the following statements are WRONG for the real function  $f(x)$  on  $(a, b)$ ?

- (A) If  $f'(c) = 0$  for some point  $c \in (a, b)$ , then  $f(x)$  has a local extrema at  $x = c$ .
- (B) If  $f(x)$  is continuous on  $(a, b)$ , then  $f(x)$  must have an absolute minimum on  $(a, b)$ .
- (C) If  $f''(x)$  exists on  $(a, b)$ , then  $f'(x)$  is continuous on  $(a, b)$ .
- (D) If  $f^2(x)$  is differentiable on  $(a, b)$ , then  $f(x)$  must be differentiable on  $(a, b)$ .

3. Consider

$$g(x) = \begin{cases} \frac{1}{x} \int_0^{\sqrt{x}} \frac{1}{t^2 + \sqrt{t} + 1} dt & , x > 0, \\ 1 & , x \leq 0. \end{cases}$$

Which of the following statements are TRUE ?

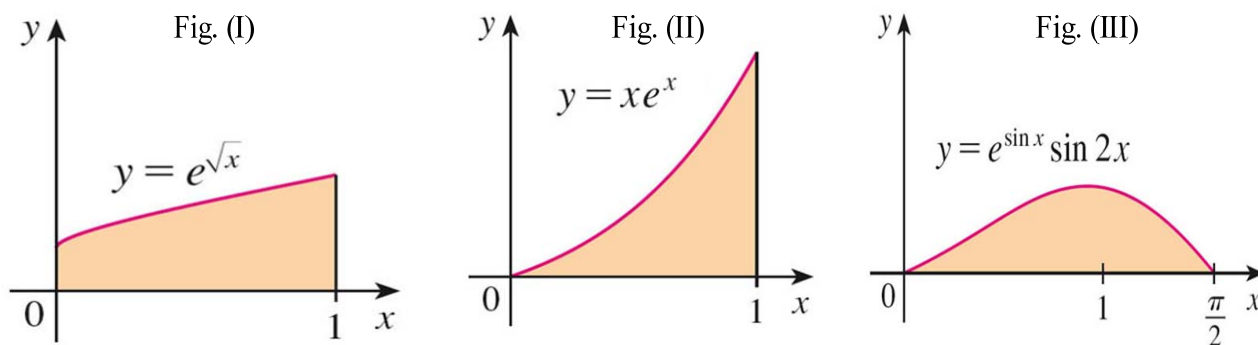
- (A)  $\lim_{x \rightarrow 0} g(x) = 1$ ;
- (B)  $\lim_{x \rightarrow \infty} g(x) = 0$ ;
- (C)  $\lim_{x \rightarrow 0} (\sin x)g(x) = 0$ ;
- (D)  $\lim_{x \rightarrow 1} g'(x) > 0$ .

4. Consider  $f(x) = \begin{cases} x^2 \sin \frac{1}{x^3} & \text{if } x > 0, \\ 0 & \text{if } x \leq 0. \end{cases}$

Which of the following statements are TRUE ?

- (A)  $f$  is continuous on  $\mathbb{R}$ ;      (B)  $f$  is differentiable on  $\mathbb{R}$ ;  
 (C)  $f'(0) = 0$ ;                      (D)  $f'$  is continuous on  $\mathbb{R}$ .

5. Which of the following statements are TRUE?



- (A) The shaded areas in Fig. (I) and Fig. (II) are equal.  
 (B) The shaded areas in Fig. (II) and Fig. (III) are equal.  
 (C) The shaded areas in Fig. (I) and Fig. (III) are equal.  
 (D) The shaded areas in Fig. (I), Fig. (II) and Fig. (III) are distinct .

◎ 填空题 (五题, 每题五分, 共二十五分, 答错不倒扣)

1. Evaluate the limit  $\lim_{x \rightarrow 0} \left( \frac{2^x + 5^x}{2} \right)^{\frac{1}{x}}$ . \_\_\_\_\_ (1)

2. Evaluate the integral  $\int \frac{x}{\sqrt{x^2 + x + 1}} dx$ . \_\_\_\_\_ (2)

3. A wedge is cut out of a circular cylinder of radius 6 by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of  $45^\circ$  along a diameter of the cylinder. Find the volume of the wedge. \_\_\_\_\_ (3)

4. Find the length of the curve  $y = \int_e^x \sqrt{(\ln t)^2 - 1} dt$  from  $x = e$  to  $x = e^2$ . \_\_\_\_\_ (4)

5. Find the length of the polar curve  $r = 1 + \sin \theta$  with  $0 \leq \theta \leq \pi$ . \_\_\_\_\_ (5)