九十九學年度第二學期微積分會考試題 (A 卷)

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

(2) 測驗時間 110 分鐘。試卷加答案卷、答案卡共計 6 頁。

(3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. $\sum_{n=1}^{\infty}\frac{3^n x^n}{(n+1)^2}$. The interval of convergence is

(A) $(-\infty,\infty)$; (B) $[-1/3,1/3]$; (C) $[-1/3,1/3)$; (D) $(-1/3,1/3)$.

2. Let $x=\cfrac{1}{4+\cfrac{1}{4+\cfrac{1}{4+\cdots}}}$. Find its value, $x=?$

(A) 1/2; (B) 1/3; (C) $-2+\sqrt{5}$; (D) does not exist.

3. Find the sum of the series $\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{2n}}{6^{2n}(2n)!}$.

(A) $\frac{1}{2}$; (B) $\frac{\sqrt{3}}{2}$; (C) $\frac{\sqrt{5}}{2}$; (D) 3.

4. Let $r(t)$ be a smooth curve in $\mathbf{R}^3$ through the point $r(t_0)$. Which of the following statements is **<u>TRUE</u>**?

(A) $r'(t_0)\times r(t_0)=0$;

(B) $r'(t_0)\cdot r(t_0)=0$;

(C) $\left(r'(t_0)\times r(t_0)\right)\times r(t_0)=0$;

(D) $\left(r'(t_0)\times r(t_0)\right)\cdot r(t_0)=0$;

5. $\lim_{(x,y)\to(0,0)} \dfrac{x^{\frac{2}{3}}\sin y}{x^2+y^2} =$

(A) -1; (B) 0; (C) 1; (D) does not exist.

6. Let $w(x,y,z)$ satisfy

$$yw + x^2w^3 + 3z^2 + z^2w - 2yz = 0 .$$

Then, $\left.\dfrac{\partial w}{\partial z}\right|_{(x,y,z)=(0,1,0)} =$

(A) 0; (B) 2; (C) 4; (D) none of these.

7. If $z = f(x,y)$, where $x = s+t$ and $y = s-t$, then $\left(\dfrac{\partial z}{\partial x}\right)^2 + \square\left(\dfrac{\partial z}{\partial y}\right)^2 = \dfrac{\partial z}{\partial s}\dfrac{\partial z}{\partial t}$.

Therefore, $\square =$

(A) -1; (B) s; (C) t; (D) does not exist.

8. The number of critical points of

$$f(x,y) = (x^2+y^2)e^{-x}$$

equals

(A) 0; (B) 1; (C) 2; (D) more than 2.

9. The value of $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{9-x^2-y^2}} e^{-(x^2+y^2+z^2)^{\frac{3}{2}}}\,dzdydx$ is

(A) $\pi(1-e^{-25})$; (B) $\dfrac{4\pi}{3}(1-e^{-27})$; (C) $\dfrac{2\pi}{3}(1-e^{-27})$; (D) $\dfrac{\pi}{3}(1-e^{-25})$.

10. The value of $\iint_R \sin(x^2+y^2)\,dA$, where $R = \left\{(x,y)\,|\,1 \le x^2+y^2 \le 4, y \ge 0\right\}$, is

(A) $\dfrac{\pi}{2}(\cos 1 - \cos 4)$; (B) $\dfrac{\pi}{2}(\sin 4 - \sin 1)$;

(C) $\pi(\cos 1 - \cos 4)$; (D) $\pi(\sin 4 - \sin 1)$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

11. Which of the following series are **<u>convergent</u>**:

(A) $\sum_{n=1}^{\infty}\frac{\sin 4n}{4^n}$.　　(B) $\sum_{n=1}^{\infty}\frac{1}{n+n\cos^2 n}$.

(C) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln n}{n}$.　　(D) $\sum_{n=1}^{\infty}\frac{1}{n^{\sin 1}}$.

12. Which of the following statements are **<u>TRUE</u>**:

(A) If $\lim_{n\to\infty} a_n = 0$, where $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive real numbers, then $\sum_{n=1}^{\infty} a_n$ is convergent.

(B) If $\sum_{n=1}^{\infty} a_n$ is conditionally convergent, then $\lim_{n\to\infty} |a_n| = 0$.

(C) If $\sum_{n=1}^{\infty} a_n$ is conditionally convergent, then there exists a rearrangement $\{b_n\}_{n=1}^{\infty}$ of $\{a_n\}_{n=1}^{\infty}$ such that $\sum_{n=1}^{\infty} b_n = \infty$.

(D) If $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then $\sum_{n=1}^{\infty} a_n$ is conditionally convergent

13. Which of the following statements are **<u>TRUE</u>** for the function

$$f(x,y)=(2x-x^2)(2y-y^2).$$

(A) $(0,0)$ is a saddle point.

(B) $f(x,y)$ has more than one saddle point.

(C) $(1,1)$ is a local maximum.

(D) $(1,1)$ is an absolute maximum.

14. Let $f(x,y)=\begin{cases}\frac{y^2-x^2}{x^2+y^2}, & \text{if } (x,y)\neq(0,0),\\ 1, & \text{if } (x,y)=(0,0).\end{cases}$

Which of the followings statements are **<u>TRUE</u>**？

(A) f is continuous at $(0,0)$,

(B) f is differentiable at $(0,y)$ for $y\neq 0$,

(C) $f_x(0,0)$ exists,

(D) $f_y(0,0)$ exists.

15. Which of the following iterate integrals are equal to the iterate integral

$$\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^x f(x,y,z)\ dzdydx$$

(A) $\int_0^1\int_0^z\int_0^{\sqrt{1-x^2}} f(x,y,z)\ dydzdx$,

(B) $\int_0^1\int_0^x\int_0^{\sqrt{1-x^2}} f(x,y,z)\ dydzdx$,

(C) $\int_0^1\int_0^{\sqrt{1-z^2}}\int_z^{\sqrt{1-z^2}} f(x,y,z)\ dxdydz$,

(D) $\int_0^1\int_0^{\sqrt{1-z^2}}\int_z^{\sqrt{1-y^2}} f(x,y,z)\ dxdydz$.

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. The Maclaulin series of $\int_0^x \frac{\arctan t}{t}dt =$ ____(1)_____.

2. The maximum of $f(x,y,z) = xy + z^2$ on the intersection of $y - x = 0$ and $x^2 + y^2 + z^2 = 4$ is ____(2)____.

3. Evaluate $\int_0^\pi\int_y^\pi \frac{\sin x}{x}\ dxdy$. ____(3)____

4. The value of $\iint_R \frac{y-x}{y+x}dA$, where R is the square with vertices $(0,2),(1,1),(2,2)$ and $(1,3)$ is ____(4)____ .

5. Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{\frac{(x^2+y^2)}{3}}$ and below the sphere $x^2 + y^2 + z^2 = z$ (see the figure below, $\phi = \frac{\pi}{3}$). ____(5)____