

### 14.3 Partial Derivatives

#### ◎ 單選擇題

1. Let  $w(x, y, z)$  satisfy

$$yw + x^2w^3 + 3z^2 + z^2w - 2yz = 0.$$

$$\text{Then, } \left. \frac{\partial w}{\partial z} \right|_{(x,y,z)=(0,1,0)} =$$

- (A) 0;      (B) 2;      (C) 4;      (D) none of these.

Ans: B [99 學年度]

2. Suppose that  $a$  and  $b$  are positive integers, such that  $f_{xy}(0,0) = -1$ , where

$$f(x, y) = \begin{cases} xy \frac{x^a - y^b}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

Which one of the following statements is **always true** ?

- (A)  $a = 2$ ;      (B)  $a = 3$ ;      (C)  $b = 2$ ;      (D)  $b = 3$ .

Ans: C [102 學年度]

3. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is differentiable, and if  $F(x, y) = y + f(x^2 - y^2)$ , then

$$y \frac{\partial F(x, y)}{\partial x} + x \frac{\partial F(x, y)}{\partial y} =$$

- (A) 0;      (B)  $x$ ;      (C)  $y$ ;      (D)  $2xyf'(x^2 - y^2)$ .

Ans: B [104 學年度]

#### ◎ 多選擇題

1. Let  $f(x, y) = \begin{cases} \frac{y^2 - x^2}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0), \\ 1, & \text{if } (x, y) = (0, 0). \end{cases}$

Which of the followings statements are **TRUE** ?

- (A)  $f$  is continuous at  $(0, 0)$ ,  
(B)  $f$  is differentiable at  $(0, y)$  for  $y \neq 0$ ,  
(C)  $f_x(0, 0)$  exists,  
(D)  $f_y(0, 0)$  exists.

Ans: BD [99 學年度]

2. Let

$$f(x, y) = \begin{cases} \frac{xy^3}{x^2 + y^6}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$

Which of the following statements are **true**?

(A)  $f$  is continuous at  $(0, 0)$ .

(B)  $f_x(0, 0) = f_y(0, 0) = 0$ .

(C)  $f_x(a, b) = \frac{b^3(b^6 - a^2)}{(a^2 + b^6)^2}$  for  $(a, b) \neq (0, 0)$ .

(D)  $f$  is differentiable at  $(0, 0)$ .

Ans: BC [100 學年度]

3. Consider the following function

$$f(x, y) = \begin{cases} \frac{(x-y)^2}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0); \\ 1, & \text{if } (x, y) = (0, 0). \end{cases}$$

Which of the following statements are **TRUE**?

(A)  $f_x(0, 0)$  and  $f_y(0, 0)$  both exist.

(B)  $f$  is continuous at  $(0, 0)$ .

(C)  $f$  is differentiable at  $(0, 0)$ .

(D)  $f$  is differentiable at  $(a, b)$  with  $(a, b) \neq (0, 0)$ .

Ans: AD [101 學年度]

1. Let  $F(x, y) = \iint_{D(x, y)} \sin(st) dA$ , where  $D(x, y) = \{(s, t) | 0 \leq s \leq x, 0 \leq t \leq 2y\}$ . Which of the following are **correct**?

(A)  $F_x(x, y) = x^{-1}(1 - \cos(xy))$  for  $x > 0$  and  $y > 0$ ;

(B)  $F_y(x, y) = y^{-1}(1 - \cos(2xy))$  for  $x > 0$  and  $y > 0$ ;

(C)  $F_x(0, 0)$  **does not** exist;

(D)  $F_{xy} = F_{yx}$  for  $x > 0$  and  $y > 0$ .

Ans: BD [105 學年度]

◎ 填充題

1. Let  $z = z(x, y)$  be function **implicitly** given by  $yz^3 + x^2z^2 = e^{xyz}$ ,  $z(0,1) = 1$ .

Then,  $\left. \frac{\partial z}{\partial x} \right|_{(x,y)=(0,1)} = \underline{\hspace{2cm}}$ .

Ans:  $\frac{1}{3}$  [100 學年度]

2. If  $z^3x + (x^2 + y)z = 0$ , then  $\left. \frac{\partial z}{\partial x} \right|_{(x,y,z)=(0,1,0)} = \underline{\hspace{2cm}}$ .

Ans: 0 [101 學年度]

3. Let  $w(x, y, z)$  satisfy  $yw + xw + 3z^2 + zw - 6xz = 0$ .

Then  $\left. \frac{\partial w}{\partial z} \right|_{(x,y,z)=(1,1,1)} = \underline{\hspace{2cm}}$ .

Ans:  $-\frac{1}{3}$  [102 學年度]

4. Consider

$$f(x, y) = \sqrt{x^2 + y^2} \cos\left(yxe^{(x^2+y^2)^{3/2}}\right).$$

Then  $f_x(-1,0) = \underline{\hspace{2cm}}$

Ans: -1 [103 學年度]

5. Let  $f(x, y) = \begin{cases} x^2 \tan^{-1} \frac{y}{x} - y^2 \tan^{-1} \frac{x}{y} & \text{if } xy \neq 0, \\ 0 & \text{if } xy = 0. \end{cases}$  Then  $f_{xy}(0,0) =$

                      
Ans: -1 [104 學年度]