14.6 Directional Derivatives and the Gradient Vector

◎ 單選擇題

1. Suppose that $f_x(a,b)$ and $f_y(a,b)$ exist, which one of the followings is **TRUE**.

   (A) $\lim_{(x,y)\to(a,b)} f(x,y)$ exists.

   (B) f is differentiable at (a,b).

   (C) $D_{\vec{u}} f(a,b)$ exists for any unit vector $\vec{u}$.

   (D) f may be discontinuous at (a,b).

Ans: D [101 學年度]

2. Suppose S is the surface of $y = g(x,z)$ where g is differentiable, and the point $P(x_0, y_0, z_0) \in S$.

   Let $G(x,y,z) = g(x,z) - y$ and $S_0 = \{(x,y,z) | G(x,y,z) = 0\}$

   Let T be the tangent plane of S at P, and T_0 be the tangent plane of S_0 at P. Which one of the followings is **correct**.

   (A) $S \neq S_0$;

   (B) The equation for T_0 is

   $$g_x(x_0,z_0)(x-x_0) - (y-y_0) + G_z(x_0,y_0,z_0)(z-z_0) = 0;$$

   (C) $T \neq T_0$;

   (D) The equation for T is

   $$G_x(x_0,y_0,z_0)(x-x_0) + g_z(x_0,z_0)(y-y_0) - (z-z_0) = 0.$$

Ans: B [101 學年度]

3. Consider

$$f(x,y)=\begin{cases}\dfrac{xy^2}{x^2+y^2}, & \text{if } (x,y)\neq(0,0);\\ 0, & \text{if } (x,y)=(0,0).\end{cases}$$

Then the directional derivative $D_u f(0,0)$, where $\mathbf{u}=\langle 1/\sqrt{2}, 1/\sqrt{2}\rangle$, equals

(A) 0; (B) $\frac{\sqrt{2}}{4}$; (C) $\frac{\sqrt{2}}{2}$; (D) $\sqrt{2}$.

Ans: B [103 學年度]

◎ 多選擇題

1. Which of the following statements are always **true** ?

(A) If $f(x,y)$ is continuous at (a,b), then $\lim\limits_{(x,y)\to(a,b)} f(x,y)=f(a,b)$.

(B) If $f_x(a,b)$ and $f_y(a,b)$ both exist, then $f(x,y)$ is continuous at (a,b).

(C) There exists a function f such that $f_x(x,y)=x^2+xy^2$, $f_y(x,y)=x^2y+2y^2$;

(D) If the partial derivatives f_x and f_y exist, then the directional derivative $D_u f(x,y)=\nabla f(x,y)\cdot u$, where $\nabla f(x,y)$ is the gradient of f at (x,y).

Ans: AC [103 學年度]

2. Let $f(x,y)=\begin{cases}\frac{x^2y}{x^4+y^2} & \text{if } (x,y)\neq(0,0),\\ 0 & \text{if } (x,y)=(0,0).\end{cases}$ Which of the following statements are **true** ?

(A) $f_x(0,0)=f_y(0,0)=0$**;**

(B) f is continuous at $(0,0)$**;**

(C) f is differentiable at $(0,0)$;

(D) $D_{\langle\frac{\sqrt{3}}{2},\frac{1}{2}\rangle}f(0,0)=\frac{3}{2}$.

Ans: AD [104 學年度]

3. For the statement: "If the directional derivative $D_{\vec{u}}f(0,0)$ exists for any unit vector $\vec{u}$, then $f(x,y)$ is continuous at the point $(0,0)$", which of the following are **counterexamples**?

(A) $f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$

(B) $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$

(C) $f(x,y) = \begin{cases} \frac{(2x^2+x^4)y}{(2x^2+x^4)^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$

(D) $f(x,y) = x^2 + y^2$.

Ans: AC [105 學年度]