

## 14.7 Maximum and Minimum Values

### ◎ 單選擇題

1. The number of critical points of

$$f(x, y) = (x^2 + y^2)e^{-x}$$

equals

- (A) 0; (B) 1; (C) 2; (D) more than 2.

Ans: C [99 學年度]

2. ]Let  $f(x, y) = x^4 - 8x^2 + 3y^2 - 6y$ . Which of the following statements is **true**?

- (A)  $f$  has an absolute maximum.  
(B)  $f$  has two saddle points.  
(C)  $f$  has two local minima.  
(D)  $f$  has two local maxima.

Ans: C [100 學年度]

3. Consider the following function

$$f(x, y) = x^3 + 2xy - y^2 + 1.$$

Which one of the following statements is **TRUE**?

- (A)  $f$  has 3 critical points.  
(B)  $f$  has a local minimum at  $\left(\frac{-2}{3}, \frac{-2}{3}\right)$ .  
(C)  $f$  has a local minimum at  $(0, 0)$ .  
(D)  $f$  has a saddle point at  $(0, 0)$ .

Ans: D [101 學年度]

4. Let  $f(x, y) = x^4 - 4xy + 8y^2 - 1$ . Which one of the following statements is **true** ?

- (A)  $f$  has 2 critical points.
- (B)  $f$  has a local minimum at (1,1) .
- (C)  $f$  has an absolute minimum at ( 1, 1) .
- (D)  $f$  has a saddle point at (0,0) .

Ans: D [102 學年度]

5. Consider the function

$$f(x, y) = -\frac{1}{4}x^4 + \frac{2}{3}x^3 + 4xy - y^2.$$

Which one of the following statements is **true** ?

- (A)  $f(x, y)$  has exactly four critical points;
- (B)  $f(x, y)$  has exactly one local minimum;
- (C)  $f(x, y)$  has exactly two local maximum;
- (D)  $f(x, y)$  has exactly two saddle points.

Ans: C [103 學年度]

6. Let  $f(x, y) = x^2 + y^2 + x^2y + 4$ . How many **saddle points** does  $f$  possess?

- (A) 0;                      (B) 1;                      (C) 2;                      (D) 3.

Ans: C [105 學年度]

◎ 多選擇題

1. Which of the following statements are **TRUE** for the function

$$f(x, y) = (2x - x^2)(2y - y^2).$$

- (A) (0,0) is a saddle point.
- (B)  $f(x, y)$  has more than one saddle point.
- (C) (1,1) is a local maximum.
- (D) (1,1) is an absolute maximum.

Ans: ABC [99 學年度]

2. Let function  $f(x, y) = \ln(x^2y^2)$  and point  $P = (1, 2)$ .

Which of the following statements are **true** ?

(A) The gradient of  $f$  at  $P$  is  $\langle 2, 1 \rangle$ .

(B) The directional derivative of  $f$  at  $P$  in the direction  $\frac{1}{\sqrt{5}}\langle 2, 1 \rangle$  is 5.

(C) The maximum rate of change of  $f$  at  $P$  occurs in the direction  $\frac{1}{\sqrt{5}}\langle 2, 1 \rangle$ .

(D) The maximum rate of change of  $f$  at  $P$  is 5.

Ans: AC [102 學年度]

3. Let  $f(x, y) = -x^3 + 4xy - y^2 + 1$ . Which of the following statements are **true** ?

(A)  $f$  has a local minimum at  $(\frac{8}{3}, \frac{16}{3})$ ;

(B)  $f$  has a local maximum at  $(0, 0)$ ;

(C)  $f$  has two critical points;

(D)  $f$  has one saddle point.

Ans: CD [104 學年度]

### ◎ 填空題

1. The **absolute maximum** of  $f(x, y) = -\frac{2y}{x^2+y^2+4}$  on

$D = \{(x, y) : x^2 + y^2 \leq 1\}$  is \_\_\_\_\_.

Ans:  $\frac{2}{5}$  [104 學年度]

2. The **absolute maximum value** of  $f(x, y, z) = xy - yz$  subject to the constraint

$x^2 + y^2 + \frac{z^2}{2} = 6$  is \_\_\_\_\_ (3) \_\_\_\_\_.

Ans:  $3\sqrt{3}$  [105 學年度]