

### 15.3 Double Integrals in Polar Coordinates

◎ 單選擇題

1. The value of  $\iint_R \sin(x^2 + y^2) dA$ , where  $R = \{(x, y) | 1 \leq x^2 + y^2 \leq 4, y \geq 0\}$ , is

- (A)  $\frac{\pi}{2}(\cos 1 - \cos 4)$ ;                      (B)  $\frac{\pi}{2}(\sin 4 - \sin 1)$ ;  
(C)  $\pi(\cos 1 - \cos 4)$ ;                      (D)  $\pi(\sin 4 - \sin 1)$ .

Ans: A [99 學年度]

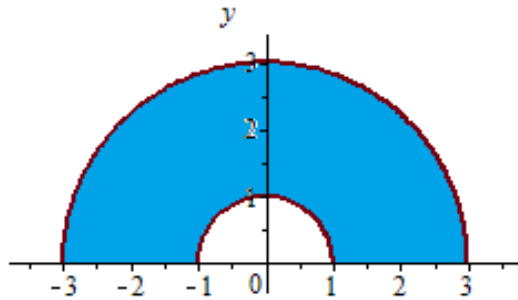
2. The integral  $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-x^2-y^2} dy dx$  is

- (A)  $\frac{\pi}{4}(1 - e^{-1})$ ,                      (B)  $\frac{\pi}{3}(1 - e^{-1})$ ,  
(C)  $\frac{\pi}{2}(1 - e^{-1})$ ,                      (D)  $\pi(1 - e^{-1})$ ,

Ans: A [100 學年度]

3. Evaluate  $\iint_R (3x + 4y^2) dA$ , where  $R$  is the region in the upper half-plane bounded by the circles  $x^2 + y^2 = 1$ ,  $x^2 + y^2 = 9$  and  $y = 0$ .

- (A)  $10\pi$ ;                      (B)  $20\pi$ ;                      (C)  $30\pi$ ;                      (D)  $40\pi$ .



Ans: D

4. The iterated integral  $\int_0^1 \int_y^1 (1 + x^2)^{-1} dx dy =$

- (A)  $\tan^{-1} 1$ ;                      (B)  $\ln 2$ ;                      (C)  $\frac{\tan^{-1} 1}{2}$ ;                      (D)  $\frac{\ln 2}{2}$ .

Ans: D [103 學年度]

5. Let  $D = \{(x, y) | x + y \leq 1, x \geq 0, y \geq 0\}$ . Then the double integral

$$\iint_D e^{x+y} dA =$$

- (A) 1;      (B) 2;      (C)  $\frac{e}{2}$ ;      (D)  $e$ .

Ans: A [103 學年度]

◎ 多選擇題

1. Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be continuous. Which of the following double integrals are **the same**?

- (A)  $\iint_R f(x, y) dA$ ,  $R$  is the region in the first quadrant bounded by the lines  $x = 2$ ,  $y = 2$  and the circle  $x^2 + y^2 = 1$ ;
- (B)  $\int_0^1 \int_{\sqrt{1-x^2}}^2 f(x, y) dy dx + \int_0^2 \int_1^2 f(x, y) dx dy$ ;
- (C)  $\int_0^2 \int_0^2 f(x, y) dy dx + \int_0^1 \int_0^{\sqrt{1-x^2}} f(x, y) dy dx$ ;
- (D)  $\int_0^{\frac{\pi}{4}} \int_1^{\frac{2}{\cos\theta}} f(r \cos \theta, r \sin \theta) r dr d\theta + \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} \int_1^{\frac{2}{\sin\theta}} f(r \cos \theta, r \sin \theta) r dr d\theta$ .

Ans: ABD [101 學年度]

2. Set  $c = \iint_D (x^2 + y^2) dA$ , where  $D = \{(x, y) | x^2 + y^2 \leq 1, x \leq y\}$ . Which of the following integrals are equal to  $c$ ?

- (A)  $\iint_{D_1} (x^2 + y^2) dA$ , where  $D_1 = \{(x, y) | x^2 + y^2 \leq 1, x \geq 0\}$ .
- (B)  $\iint_{D_2} (x^2 + y^2) dA$ , where  $D_2 = \{(x, y) | x^2 + y^2 \leq 1, y \geq 0\}$ .
- (C)  $\iint_{D_3} x^2 dA$ , where  $D_3 = [0, 1] \times \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ .
- (D)  $\iint_{D_4} x^3 dA$ , where  $D_4 = [0, 1] \times \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ .

Ans: ABD

◎ 填充題

1. Evaluate  $\int_0^\pi \int_y^\pi \frac{\sin x}{x} dx dy$ . \_\_\_\_\_

Ans: 2 [99 學年度]

2. Evaluate the integral  $\iint_R \frac{\sin y}{y} dA$ , where  $R = \{(x, y) | 0 \leq x \leq \pi, x \leq y \leq \pi\}$ .

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Ans: 2 [101 學年度]