15.8 Triple Integrals in Spherical Coordinates

◎ 單選擇題

1. The value of $\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{9-x^2-y^2}} e^{-(x^2+y^2+z^2)^{\frac{3}{2}}}\,dzdydx$ is

   (A) $\pi(1-e^{-25})$; (B) $\frac{4\pi}{3}(1-e^{-27})$; (C) $\frac{2\pi}{3}(1-e^{-27})$; (D) $\frac{\pi}{3}(1-e^{-25})$.

Ans: C [99 學年度]

2. The volume of the solid that lies above the cone $z=\sqrt{3x^2+3y^2}$ and below the sphere $x^2+y^2+z^2=z$ is

   (A) $\frac{7\pi}{96}$; (B) $\frac{3\pi}{16}$; (C) $\frac{\pi}{32}$; (D) $\frac{\pi}{96}$.

Ans: A [102 學年度]

3. Let $E=\{(x,y,z)|1\le x^2+y^2+z^2\le 4\}$. Then the triple integral

$$\iiint_E \sin[(x^2+y^2+z^2)^{\frac{3}{2}}]\,dV=$$

   (A) $4\pi(\cos 1-\cos 8)$;

   (B) $\left(\frac{\pi}{2}\right)^2(\cos 1-\cos 4)$;

   (C) $\frac{4\pi}{3}(\cos 1-\cos 8)$;

   (D) $\frac{\pi^2}{3}(\cos 1-\cos 4)$.

Ans: C [103 學年度]

◎ 多選擇題

1. The integral $\iiint_E 1dV$, where E is the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\le 1$, is

(A) $\iiint_B abcdV$ where B is the unit ball.

(B) $\frac{\pi abc}{3}$.

(C) $\int_0^1\int_0^{2\pi}\int_0^{\pi} abc\rho^2\sin\phi d\phi d\theta d\rho$.

(D) $\frac{4\pi abc}{3}$.

Ans: ACD [100 學年度]

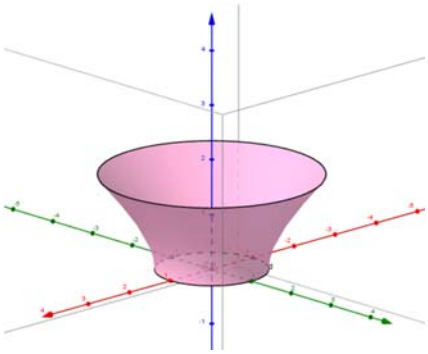
Which of the following iterated integrals are equal to the triple integral $\iiint_E f(x,y,z)dV$? Where E is the solid bounded by the hyperboloid of one sheet $x^2+y^2-z^2=1$ and planes $z=\sqrt{3}$ and $z=0$.

(A) $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2-1}}^{\sqrt{3}} f(x,y,z)\,dzdydx$;

(B) $\int_0^{\sqrt{3}}\int_{-\sqrt{z^2+1}}^{\sqrt{z^2+1}}\int_{-\sqrt{z^2-x^2+1}}^{\sqrt{z^2-x^2+1}} f(x,y,z)\,dydxdz$;

(C) $\int_0^{\sqrt{3}}\int_{-\sqrt{z^2+1}}^{\sqrt{z^2+1}}\int_{-\sqrt{z^2-y^2+1}}^{\sqrt{z^2-y^2+1}} f(x,y,z)\,dxdydz$;

(D) $\int_0^{2\pi}\int_0^1\int_{\sqrt{r^2-1}}^{\sqrt{3}} f(r\cos\theta,r\sin\theta,z)r\,dzdrd\theta$.

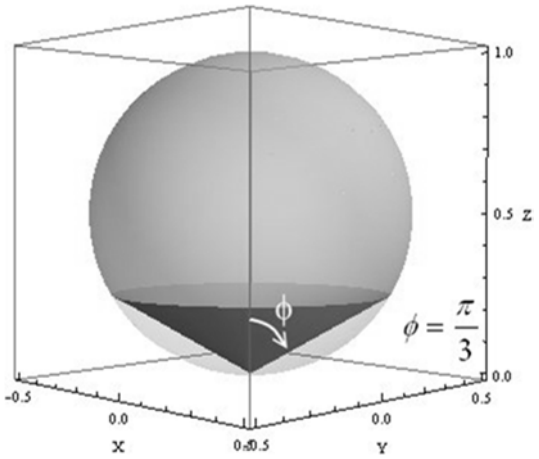


Ans: BC [104 學年度]

◎ 填空題

1. Use spherical coordinates to find the volume of the solid that lies above the cone $z=\sqrt{\frac{(x^2+y^2)}{3}}$ and below the sphere $x^2+y^2+z^2=z$ (see the figure below, $\phi=\frac{\pi}{3}$). ____(5)____

Ans: $\frac{5\pi}{32}$ [99 學年度]



2. The **volume** of the solid that lies within the sphere $x^2+y^2+z^2=4$, above the *xy*-plane, and below the cone $z=\sqrt{x^2+y^2}$ is _____.

Ans: $\frac{8\sqrt{2}}{3}\pi$ [104 學年度]