15.9 Change of Variables in Multiple Integrals

◎ 單選擇題

1. The volume of the solid $\{(x,y,z)|(x+y)^2+(y+z)^2+(z+x)^2 \le 1\}$ is

(A) $\frac{\pi}{3}$, (B) $\frac{2\pi}{3}$, (C) π, (D) $\frac{4\pi}{3}$.

Ans: B [102 學年度]

2. E be a region in $\mathbb{R}^3$ for which the value of the triple integral

$$\iiint_E (1-x^2-2y^2-3z^2)dV$$

is a **maximum** . Then E is

(A) $\left\{(x,y,z)\middle|-1 \le x \le 1, -\frac{1}{\sqrt{2}} \le y \le \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{3}} \le z \le \frac{1}{\sqrt{3}}\right\}$;

(B) $\left\{(x,y,z)\middle|-\sqrt{1-2y^2-3z^2} \le x \le \sqrt{1-2y^2-3z^2}, 2y^2+3z^2 \le 1\right\}$;

(C) $\{(\rho,\theta,\emptyset)|0 \le \rho \le 1, 0 \le \theta \le 2\pi, 0 \le \emptyset \le \pi\}$;

(D) $\left\{(r,\theta,z)\middle|0 \le r \le 1, 0 \le \theta \le 2\pi, -\frac{1}{\sqrt{3}} \le z \le \frac{1}{\sqrt{3}}\right\}$.

Ans: B [103 學年度]

3. The **area** of the region $\{(x,y)|(x+y)^2+|x-y| \le 1\}$ is

(A) $\frac{2}{3}$; (B) $\frac{4}{3}$; (C) 2; (D) $\frac{8}{3}$.

Ans: B 03 學年度]

4. The value of the double integral $\iint_D \frac{x-y}{x+y}\, dA$, where D is the square with verties $(0,2), (1,1), (2,2)$ and $(1,3)$, is

(A) $-\ln(2)$; (B) $-2\ln(2)$; (C) $3\ln(2)$; (D) $6\ln(2)$.

Ans: A [104 學年度]

5. The iterated integral $\int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2} dydx$ =

(A) $\frac{\pi}{2}$; (B) $\frac{\pi}{4}$; (C) $1+\frac{\pi}{2}$; (D) $1+\frac{\pi}{4}$.

Ans: C [105 學年度]

6. The triple integral $\iiint_{\mathbb{R}^3} \frac{dV}{(x^2+y^2+z^2+1)^2} =$

(A) π; (B) π^2; (C) π^3; (D) π^4.

Ans: B [105 學年度]

◎ 填空題

1. Let R be the square with vertices (0,0), $(1,1)$, $(1,-1)$ and (2,0). Then

$$\iint_R xydA =$$

_____

Ans: 0 [103 學年度]

2. double integral

$$\int\int_R (x-y)^2 \sin^2(x+y)\, dA = \_\_\_\_\_$$

where R is the parallelogram with successive vertices $(\pi,0), (2\pi,\pi), (\pi,2\pi)$ and $(0,\pi)$.

Ans: $\frac{\pi^4}{3}$ [105 學年度]