Ch12-4

單選題

■ Let $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ be different vectors. The notations × and · denote the cross product and inner product, respectively. Which of the following is not true ?

(a) $\boldsymbol{a} \times (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{a} \times \boldsymbol{c}$.

(b) $(\boldsymbol{a} + \boldsymbol{b}) \times \boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{c} + \boldsymbol{b} \times \boldsymbol{c}$.

(c) $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \times \boldsymbol{b}) \cdot \boldsymbol{c}$.

(d) $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{b} \times \boldsymbol{a}$.

Ans：d

SOL：

皆為課本定理（課本 790 頁）

$\boldsymbol{a} \times \boldsymbol{b} = - \boldsymbol{b} \times \boldsymbol{a}$.

填空題

■ Let a, b and c be vectors in R^3. Suppose that $a \bullet (b \times c) = 3$. Then $(a \times b) \bullet c =$ _____ .

Ans：3

SOL：

$a \bullet (b \times c) = (a \times b) \bullet c = 3$ （課本 790 頁）

■ Consider $\vec{A}=\vec{i}+\vec{j}+\vec{k}$, $\vec{B}=\vec{i}$, $\vec{C}=\vec{i}+2\vec{j}+w\vec{k}$, find w to make the three vector coplanar .

w = _____(3)_____.

Ans：2

SOL：

先對$\vec{A}, \vec{B}$兩向量做外積找出與$\vec{A}, \vec{B}$兩向量都垂直的向量

$$\vec{A}\times\vec{B}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 1 & 1\\ 1 & 0 & 0\end{vmatrix}=\vec{j}-\vec{k}$$

因為$\vec{C}$與$\vec{A}, \vec{B}$共面，所以此向量與向量$\vec{C}$做內積要等於 0

$$\left(\vec{j}-\vec{k}\right)\bullet\vec{C}=0\Rightarrow\left(\vec{j}-\vec{k}\right)\bullet\left(\vec{i}+2\vec{j}+w\vec{k}\right)=2-w=0$$

$$\Rightarrow w=2$$

■ $\vec{a}=\langle 1.111, 2.222, -3.333\rangle$, $\vec{b}=\langle 3.333, 4.444, 5.555\rangle$, $\vec{c}=\langle -2.222, 2.222, 4.$

$\vec{d}=\vec{b}-\vec{c}$ and $\vec{e}=\vec{a}-\vec{c}$. *Evaluate* $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{d}\times\vec{e}$ = _____(5)___.

Ans：0

SOL：

$$\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\left(\vec{b}-\vec{c}\right)\times\left(\vec{a}-\vec{c}\right)$$
$$=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{b}\times\left(\vec{a}-\vec{c}\right)-\vec{c}\times\left(\vec{a}-\vec{c}\right)$$
$$=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{b}\times\vec{a}-\vec{b}\times\vec{c}-\vec{c}\times\vec{a}-\vec{c}\times\vec{c}$$
$$=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\left(-\vec{a}\times\vec{b}\right)-\vec{b}\times\vec{c}-\vec{c}\times\vec{a}-0=0$$

Since $\vec{c}\times\vec{c}=\left|\vec{c}\right|\cdot\left|\vec{c}\right|\cdot\sin(0)=0$