Ch13.2

單選題

■ If $\vec{F}(t)=\hat{i}+t\hat{j}+t^2\hat{k}$

$\vec{G}(t)=t\hat{i}+e^t\hat{j}+3\hat{k}$

Which of the following is not true

(A) $\vec{F}\,'(t)=\hat{j}+2t\hat{k}$;

(B) $G\,'(t)\times\vec{F}(t)=(t^2e^t)\hat{i}+t^2\hat{j}+(t-e^t)\hat{k}$;

(C) $(\vec{F}\times\vec{G})'(t)=(3-2te^t-t^2e^t)\hat{i}+(3t^2)\hat{j}+(e^t-2t)\hat{k}$;

(D) $(\vec{F}\times\vec{G})'(t)=\vec{F}\,'(t)\times\vec{G}(t)+\vec{F}(t)\times\vec{G}\,'(t)$.

■ Ans：B

Sol：

A) $\vec{F}\,'(t)=\hat{j}+2t\hat{k}$

B) $\vec{G}'(t)=\hat{i}+e^t\hat{j},\vec{G}'\times\vec{F}=\begin{vmatrix} i & j & k \\ 1 & t & t^2 \\ 1 & e^t & 0 \end{vmatrix}=(-t^2e^t)\hat{i}+t^2\hat{j}+(e^t-t)\hat{k}$ $\quad\therefore B$ false

C) $(\vec{F}\times\vec{G})(t)=(3t-t^2e^t)\hat{i}+(t^3-3)\hat{j}+(e^t-t^2)\hat{k}$,

so $(\vec{F}\times\vec{G})'(t)=(3-2te^t-t^2e^t)\hat{i}+(3t^2)\hat{j}+(e^t-2t)\hat{k}$

D) $(\vec{F}\times\vec{G})'(t)=\vec{F}\,'(t)\times\vec{G}(t)+\vec{F}(t)\times\vec{G}\,'(t)$ by Thm of p.826

■ Let $r(t)$ be a smooth curve in $\mathbf{R}^3$ through the point $r(t_0)$. Which of the following statements is **TRUE**?

(A) $r'(t_0)\times r(t_0)=0$;

(B) $r'(t_0)\cdot r(t_0)=0$;

(C) $\left(r'(t_0)\times r(t_0)\right)\times r(t_0)=0$;

(D) $\left(r'(t_0)\times r(t_0)\right)\cdot r(t_0)=0$

Ans：D

;

Sol:

根據課本 13-2(p.824)圖一

$r'(t_0)$是在t_0的切線

$r(t_0)$是原點到t_0的直線

可知(A)為錯因為兩者不一定平行

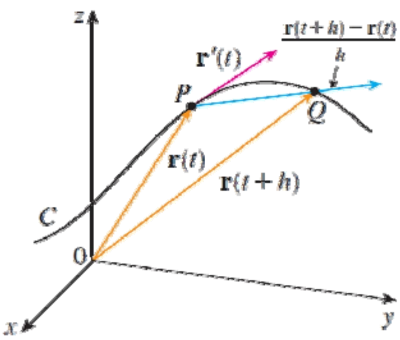
(B)為錯因為兩者不一定垂直

(C)為錯因為$r'(t_0)\times r(t_0)$是垂直於$r'(t_0)\times r(t_0)$所構成的平面的向量

所以兩垂直向量的外積不為0

(D)為對因為因為$r'(t_0)\times r(t_0)$是垂直於$r'(t_0)\times r(t_0)$所構成的平面的向量

所以內積為0



(b) The tangent vector

填充題

■ Let $f(x,y)=x-y^2$. Find the tangent line to the level curve $f(x,y)=2$ at the point $(3,-1)$. ___(7)___ Ans： $x+2y=1$

Sol:

$let \quad y=t \Rightarrow x=2+t^2$

$r(t)=(2+t^2,t,0) \Rightarrow r'(t)=(2t,1,0)$

由(3-1)可知t = -1代入

$r'(-1)=(-2,1,0) \Rightarrow x=-2t+3, y=t-1$

$\Rightarrow x+2y=-2t+3+2(t-1)=1$