Ch13-4

填空題

■ For $r(t)=t^2\vec{i}+t\vec{j}$, find a_T and a_N , the tangential and normal components of acceleration.

Ans： $a_T=\dfrac{4t}{\sqrt{4t^2+1}}$; $a_N=\dfrac{2}{\sqrt{4t^2+1}}$

Sol:

$$a_T=\frac{r'(t)\cdot r''(t)}{|r'(t)|}=\frac{(2t\vec{i}+\vec{j})\cdot(2\vec{i})}{|(2t\vec{i}+\vec{j})|}=\frac{4t}{\sqrt{4t^2+1}}$$

Since $r'(t)\times r''(t)=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2t & 1 & 0\\ 2 & 0 & 0\end{vmatrix}=-2\vec{k}$

$$a_N=\frac{|r'(t)\times r''(t)|}{|r'(t)|}=\frac{|-2\vec{k}|}{|(2t\vec{i}+\vec{j})|}=\frac{2}{\sqrt{4t^2+1}}$$

■ Find the normal component of acceleration of

$\vec{r}(t)=(\sin t-t\cos t)\vec{i}+(\cos t+t\sin t)\vec{j}+t^2\vec{k}$.

$a_n=$ ______(2)______.

Ans：t

Sol:

$\vec{r}'(t)=(t\sin t)\vec{i}+(t\cos t)\vec{j}+2t\vec{k}.$

$\vec{r}''(t)=(\sin t+t\cos t)\vec{i}+(\cos t-t\sin t)\vec{j}+2\vec{k}.$

Since $r'(t)\times r''(t)=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ t\sin t & t\cos t & 2t\\ \sin t+t\cos t & \cos t-t\sin t & 2\end{vmatrix}=2t^2\sin t\vec{i}+2t^2\cos t\vec{j}-t^2\vec{k}$

$$a_N=\frac{|r'(t)\times r''(t)|}{|r'(t)|}=\frac{\sqrt{4t^4\sin^2 t+4t^4\cos^2 t+t^4}}{\sqrt{t^2\sin^2 t+t^2\cos^2 t+4t^2}}=\frac{\sqrt{5t^4}}{\sqrt{5t^2}}=t$$

■ Let the position function of a particle be $\mathbf{r}(t)=\left\langle t^2,1-2t,t\right\rangle$. Find the value of t at which its speed is smallest.

Ans：0

Sol:

$\mathrm{r}'(t)=\langle 2t,-2,1\rangle$ and $|\mathrm{r}'(t)|=\sqrt{4t^2+5}$

可以看出當 $t=0$ 時 $|\mathrm{r}'(t)|=\sqrt{5}$ 最小

■ A moving particle starts at an initial position $r(0)=\mathbf{i}+\mathbf{k}$ with initial velocity $V(0)=\mathbf{j}$. Its acceleration is $a(t)=t\mathbf{i}+e^t\mathbf{j}+\sin t\mathbf{k}$. Find its position at time t.

Ans： $r(t)=\left(\frac{t^3}{6}+1\right)\mathrm{i}+\left(e^t-1\right)\mathrm{j}+\left(-\sin t+t+1\right)\mathrm{k}$

Sol:

$$V(t)=\int t\mathbf{i}+e^t\mathbf{j}+\sin t\mathbf{k}\,dt=\frac{t^2}{2}\mathbf{i}+e^t\mathbf{j}-\cos t\mathbf{k}+\mathbf{C}$$

and $\mathbf{j}=V(0)=\mathbf{j}-\mathbf{k}+\mathbf{C}$, so $\mathbf{C}=\mathbf{k}$

$$r(t)=\int\frac{t^2}{2}\mathbf{i}+e^t\mathbf{j}+(1-\cos t)\mathbf{k}\,dt=\frac{t^3}{6}\mathbf{i}+e^t\mathbf{j}+(\mathrm{t}-\sin t)\mathbf{k}+\mathbf{D}$$

and $\mathbf{i}+\mathbf{k}=r(0)=\mathbf{j}+\mathbf{D}$, so $\mathbf{D}=\mathbf{i}-\mathbf{j}+\mathbf{k}$ and

$$r(t)=\left(1+\frac{t^3}{6}\right)\mathbf{i}+\left(e^t-1\right)\mathbf{j}+(1+\mathrm{t}-\sin t)\mathbf{k}$$