2.4 The Precise Definition of a Limit

單選題

1. Consider $f(x)=\begin{cases}-5x+2 & \text{if } x\ge 0,\\ \dfrac{-(x+1)^2}{4} & \text{if } x<0.\end{cases}$

When using the $\varepsilon-\delta$ definition to prove that $\lim\limits_{x\to -1} f(x)=0$, the largest δ for $\varepsilon=1$ is

(A) 2; (B) 1; (C) $1\frac{2}{5}$; (D) $1\frac{3}{5}$.

Ans: B [99 學年度]

2. Let $A=\{0.6,0.7,0.8,0.9\}$.

Find the **largest** number, δ, in A such that

$\left|\sqrt{4x+5}-3\right|<0.6$, whenever $|x-1|<\delta$.

(A) $\delta=0.6$, (B) $\delta=0.7$, (C) $\delta=0.8$, (D) $\delta=0.9$.

Ans: C [100 學年度]

3. Find the **maximum** positive number δ such that: If $0<|x|<\delta$ then

$\left|e^{-x}-1\right|<\frac{1}{2}$.

(A) $\ln 3$; (B) $\ln 2$; (C) $\ln\frac{3}{2}$; (D) $\ln\frac{2}{3}$.

Ans: C [102 學年度]

多選題

1. Which of the following number δ can take such that

If $0<|x-2|<\delta$, then $\left|\sqrt{x}-\sqrt{2}\right|<1$,

(A) $2\sqrt{2}+1$; (B) $2\sqrt{2}-1$; (C) 1; (D) 2.

Ans: BC [101 學年度]

2. Let $f(x) = x^2$. Then f is continuous and so $\lim_{x \to a} f(x) = a^2$ for every $a \in$ R. This means that for every $\epsilon > 0$, there exists a $\delta > 0$ such that

$$|f(x) - a^2| < \epsilon,$$

whenever

(A) $|x - a| < \delta$; (B) $0 < |x - a| < \delta$;

(C) $|x - a^2| < \delta$; (D) $0 < |x - a^2| < \delta$.

Ans: AB [103 學年度]