

5.3 The Fundamental Theorem of Calculus

單選題

1. Let $I = \lim_{x \rightarrow 0^+} \frac{1}{x} \left(\int_{1-x}^1 \frac{1}{t^3 + t} dt \right)$. Which of the following statement is **true** ?

- (A) $I = -1$. (B) $I = 1$. (C) $I = -\frac{1}{2}$. (D) $I = \frac{1}{2}$.

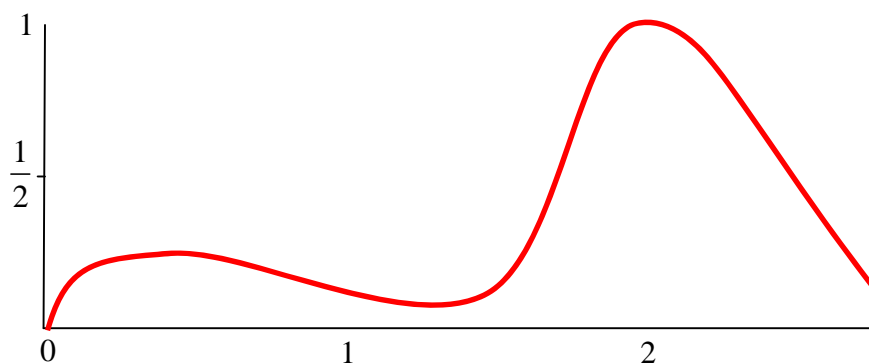
Ans: D [100 學年度]

2. Evaluate $\lim_{x \rightarrow 0} \frac{\int_0^x \sin t^2 dt}{x^2} =$

- (A) 1; (B) $\frac{1}{2}$; (C) 0; (D) ∞ .

Ans: C [101 學年度]

3. The graph of a differentiable function f is shown below :



List the following integrals in **order** from the **smallest** to the **largest**

$$\text{I} = \int_0^1 f(x) dx \quad \text{II} = \int_0^2 f(x) dx \quad \text{III} = \int_0^1 f^2(x) dx \quad \text{IV} = \int_0^2 f'(x) dx$$

- (A) $\text{I} < \text{II} < \text{III} < \text{IV}$, (B) $\text{II} < \text{III} < \text{I} < \text{IV}$,
 (C) $\text{III} < \text{I} < \text{II} < \text{IV}$, (D) $\text{IV} < \text{III} < \text{I} < \text{II}$.

Ans: C [100 學年度]

4. Evaluate the limit $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sin \frac{1}{t} dt}{x}$.

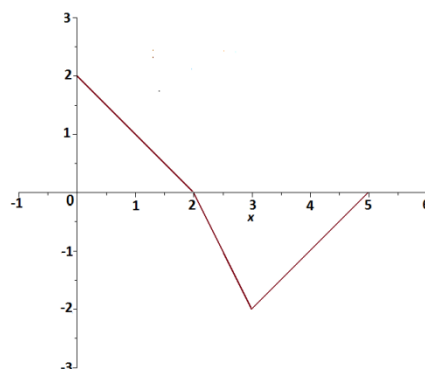
- (A) 0; (B) 1; (C) 2; (D) ∞ .

Ans: A [102 學年度]

5. Let f be the function where graph is given below and let $F(x) = \int_3^x f(t)dt$.

Which of the following values is **largest**?

- (A) $F(0)$; (B) $F(1)$;
 (C) $F(2)$; (D) $F(3)$.



Ans: C [103 學年度]

6. Let $F(x) = \int_{x^3}^1 \sqrt{2^t + 14} dt$. Then the derivative $(F^{-1})'(0) =$

- (A) $-\frac{1}{12}$; (B) $-\frac{1}{2}$; (C) 0; (D) $\frac{1}{4}$.

Ans: A [104 學年度]

7. $\lim_{n \rightarrow \infty} \left\{ \frac{1}{n^3 \sqrt{4n^2 + 1}} + \frac{8}{n^3 \sqrt{4n^2 + 4}} + \dots + \frac{n^3}{n^3 \sqrt{4n^2 + n^2}} \right\} =$

- (A) $3\sqrt{5}$; (B) $\frac{3}{2}\sqrt{5}$; (C) $-\frac{14}{3}\sqrt{5} + \frac{32}{3}$; (D) $-\frac{7}{3}\sqrt{5} + \frac{16}{3}$.

Ans: D [104 學年度]

多選題

1. Consider

$$g(x) = \begin{cases} \frac{1}{x} \int_0^{\sqrt{x}} \frac{1}{t^2 + \sqrt{t} + 1} dt & , x > 0, \\ 1 & , x \leq 0. \end{cases}$$

Which of the following statements are TRUE ?

- (A) $\lim_{x \rightarrow 0} g(x) = 1$; (B) $\lim_{x \rightarrow \infty} g(x) = 0$;
 (C) $\lim_{x \rightarrow 0} (\sin x)g(x) = 0$; (D) $\lim_{x \rightarrow 1} g'(x) > 0$.

Ans: BC [99 學年度]

填充題

1. The **derivative** of $\int_x^{x^3} \sin(t^2) dt$ is _____.

Ans: $3x^2 \sin(x^6) - \sin(x^2)$ [104 學年度]