7.3 Trigonometric Substitution

單選題

1. $\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x^2}}dx =$

(A) $\frac{\pi}{12}$; (B) $\frac{\pi}{12}-\frac{\sqrt{3}}{8}$; (C) $\frac{\sqrt{3}}{8}-\frac{\pi}{12}$; (D) $\frac{\sqrt{3}}{8}$.

Ans: B [100 學年度]

2. Evaluate the definite integral $\int_0^{\frac{\pi}{4}} x\tan^2 x dx =$

(A) $\frac{\pi}{4}-\frac{\pi^2}{32}-\ln\sqrt{2}$;

(B) $\frac{\pi}{4}-\frac{\pi^2}{32}+\ln\sqrt{2}$;

(C) $\frac{\pi}{4}-\frac{\pi^2}{16}-\ln\sqrt{2}$;

(D) $\frac{\pi}{4}-\frac{\pi^2}{16}+\ln\sqrt{2}$.

Ans: A [101 學年度]

填充題

1. Evaluate the integral $\int \frac{x}{\sqrt{x^2+x+1}}dx$. _____

Ans: $\sqrt{x^2+x+1}-\frac{1}{2}\ln\left|\frac{2}{\sqrt{3}}\sqrt{x^2+x+1}+\frac{2}{\sqrt{3}}\left(x+\frac{1}{2}\right)\right|+C$

$= \sqrt{x^2+x+1}-\frac{1}{2}\ln\left|\frac{2}{\sqrt{3}}\left[\sqrt{x^2+x+1}+\left(x+\frac{1}{2}\right)\right]\right|+C$

$=\sqrt{x^2+x+1}-\frac{1}{2}\ln\frac{2}{\sqrt{3}}-\frac{1}{2}\ln\left(\sqrt{x^2+x+1}+x+\frac{1}{2}\right)+C$

$=\sqrt{x^2+x+1}-\frac{1}{2}\ln(\sqrt{x^2+x+1}+x+\frac{1}{2})+C$ [99 學年度]

2. $\int \frac{1}{\left(x^2+1\right)^2}dx =$ _____.

Ans: $\frac{1}{2}\tan^{-1}x+\frac{1}{2}\frac{x}{x^2+1}+C$ [101 學年度]