

## 7.8 Improper Integrals

### 單選題

1. Evaluate the integral  $\int_e^{\infty} \frac{1+e^{-x^2}}{x} dx$ .

- (A)  $\frac{1}{e}$ ;      (B) 1;      (C)  $e^{-e^2}$ ;      (D) divergent.

Ans: D [99 學年度]

2. Which improper integral is **convergent**?

- (A)  $\int_0^{\pi} \tan x dx$ ;      (B)  $\int_{-\infty}^{\infty} \tan^{-1} x dx$ ;  
(C)  $\int_2^{\infty} \frac{1}{x \ln x} dx$ ;      (D)  $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$ .

Ans: D [101 學年度]

3. The improper integral  $\int_0^{\infty} x^2 e^{-x} dx =$

- (A)  $-5$ ;      (B) 1;      (C) 2;      (D)  $+\infty$ .

Ans: C [102 學年度]

### 多選題

1. Which of the following improper integrals are **convergent** ?

- (A)  $\int_1^{\infty} \frac{1}{x} dx$ .      (B)  $\int_0^{\infty} \frac{1}{x^2+1} dx$ .      (C)  $\int_1^{\infty} e^{-x} dx$ .      (D)  $\int_{-\infty}^{\infty} \frac{1}{x^4+x^2+1} dx$ .

Ans: BCD [100 學年度]

2. Suppose that  $f$  and  $g$  are continuous functions on  $\mathbb{R}$ . Which of the following statements are **true**?

(A)  $\int_{-\infty}^{\infty} f(x) dx = \lim_{t \rightarrow \infty} \int_{-t}^t f(x) dx$ .

(B) If  $\lim_{x \rightarrow \infty} f(x) = 0$ , then  $\int_1^{\infty} f(x) dx$  is convergent.

(C) If  $f$  is an even function and  $\int_{-\infty}^{\infty} f(x) dx$  is convergent, then

$$\int_{-\infty}^{\infty} f(x) dx = 2 \int_0^{\infty} f(x) dx.$$

(D) If  $0 \leq f(x) \leq g(x)$  for all  $x \in \mathbb{R}$  and  $\int_{-\infty}^{\infty} f(x) dx$  diverges, then

$$\int_{-\infty}^{\infty} g(x) dx \text{ diverges.}$$

Ans: CD [101 學年度]

3. For which values of  $a$  does the improper integral  $\int_0^{\infty} \frac{x^a}{1+x^2} dx$  converge?

- (A)  $a = -1$ ;      (B)  $a = -\frac{1}{2}$ ;      (C)  $a = \frac{1}{2}$ ;      (D)  $a = 1$ .

Ans: BC [103 學年度]

4. The improper integral

$$\int_0^{\infty} \frac{\sqrt{x}}{1+x^a} dx$$

converges for

- (A)  $a = \frac{1}{2}$ ;      (B)  $a = \frac{3}{2}$ ;      (C)  $a = \frac{5}{2}$ ;      (D)  $a = \frac{7}{2}$ .

Ans: CD [104 學年度]

5. Which of the following statements are **True** ?

- (A)  $\int_{-\infty}^{\infty} x^3 dx = 0$ ;
- (B)  $\int_3^{\infty} \frac{1}{x^{\csc 1}} dx$  is convergent;
- (C)  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan(x^3)}{1+x^2} dx = \sqrt{2}\pi$ ;
- (D)  $\int_0^{\frac{\pi}{4}} \tan^2 x dx = 1 - \frac{\pi}{4}$ .

Ans: BD [104 學年度]

### 填充題

1. Set  $f(x) = \frac{1}{x[\ln(x+1)]^p}$ . Determine **all positive values** of  $p$  for which the

improper integral  $\int_1^{\infty} f(x) dx$  **converges**. \_\_\_\_\_.

Ans:  $p > 1$  (or  $(1, \infty)$ ) [102 學年度]