8.2 Area of a Surface of Revolution

單選題

1. Find the surface area of the solid of revolution formed by rotating $y = x^4$ over $[0,3]$ about the *x*-axis.

   (A) $\int_0^3 2\pi x\sqrt{1+16x^6}\,dx$; (B) $\int_0^3 2\pi x\sqrt{1+x^3}\,dx$;

   (C) $\int_0^3 2\pi x^4\sqrt{1+16x^6}\,dx$; (D) $\int_0^3 2\pi x^4\sqrt{1+x^3}\,dx$.

Ans: C [99 學年度]

2. Find the **area** of the surface generated by revolving the curves $x = \frac{e^y + e^{-y}}{2}$, $0 \le y \le \ln 2$, about the *y*-axis.

   (A) $\frac{9}{16}\pi$; (B) $\frac{9}{8}\pi$; (C) $\pi\left(\frac{15}{16}+\ln 2\right)$; (D) $\pi\left(\frac{15}{8}+2\ln 2\right)$.

Ans: C [103 學年度]

3. The **surface area** of the figure obtained by rotating $y = x^3,\ 0 \le x \le 1$ about the *x*-axis is

   (A) $\frac{\pi}{27}\left((\sqrt{10})^3 - 1\right)$; (B) $\frac{\pi}{12}\left((\sqrt{10})^3 - 1\right)$;

   (C) $\frac{\pi}{27}\left((\sqrt{2})^3 - 1\right)$; (D) $\frac{\pi}{12}\left((\sqrt{2})^3 - 1\right)$.

Ans: A [104 學年度]

多選題

1. Which of the following statements are **true**?

(A) The smooth curve $y=f(x)$, $1\le x\le 2$, where $f(x)\le 3$, is rotated about the horizontal line $y=3$, then the area of the resulting surface is

$$\int_1^2 2\pi\left(3-f(x)\right)\sqrt{1+\left(f'(x)\right)^2}\,dx\,.$$

(B) The curve $y=x^2$, $0\le x\le 1$, is rotated about the vertical line $x=-2$, then the area of the resulting surface is $\int_0^1 2\pi\left(2+x\right)\sqrt{1+4x^2}\,dx$.

(C) The arclength of the curve $y=\ln\left(x^2+1\right), 1\le x\le e$, is P, and the arclength of the curve $y=2\ln x$, $1\le x\le e$, is Q, then $P>Q$.

(D) Suppose $f(x)\ge g(x)$, $a\le x\le b$.

Let $S=$ the arclength of the smooth curve $y=f(x)$ on $[a,b]$, and $T=$ the arclength of the smooth curve $y=g(x)$ on $[a,b]$, then $S\ge T$.

Ans: AB [101 學年度]