九十九學年度第一學期微積分會考試題參考詳解

1. Let $f(x) = 2^x - x^2$, $\forall\, x \in \mathbb{R}$. Then (c, c^2) is a point of intersection of the original two functions if and only if $f(c) = 0$. By mean value theorem, since $f''(x) = (\ln 2)^2 \cdot 2^x - 2 = 0$ has exactly one solution, $f'(x) = 0$ has at most two distinct solutions, and so on, $f(x) = 0$ has at most three distinct solutions.

   Now, by intermediate value theorem and $f(-1) = -0.5 < 0 < f(0) = 1$, $f(2) = f(4) = 0$, we know that $f(x) = 0$ has exactly three roots $c, 2, 4$, where $-1 < c < 0$.

   `Ans: D`

2.

$$
\begin{aligned}
&|f(x) - 0| < 1 \;\Leftrightarrow\; -1 < f(x) < 1 \\
\Leftrightarrow\;& (x \geq 0 \text{ and } -1 < -5x + 2 < 1) \\
\text{or }& (x < 0 \text{ and } -1 < -\frac{(x+1)^2}{4} < 1) \\
\Leftrightarrow\;& (\frac{1}{5} < x < \frac{3}{5}) \text{ or } (-3 < x < 0).
\end{aligned}
$$

   Since the minimal distance of $\frac{1}{5}$, $\frac{3}{5}$, -3, and 0 to -1, respectively, is $|0 - (-1)| = 1$, the largest δ is 1.

   `Ans: B`

3.

$$
\begin{aligned}
&\lim_{x\to 0}\left(\frac{1}{\ln(x+1)} - \frac{x+1}{x}\right) = \lim_{x\to 0}\frac{x - (x+1)\ln(x+1)}{x\ln(x+1)} \\
\overset{l'H}{=}\;& \lim_{x\to 0}\frac{1 - \ln(x+1) - 1}{\ln(x+1) + \frac{x}{x+1}} = \lim_{x\to 0}\frac{-(x+1)\ln(x+1)}{(x+1)\ln(x+1) + x}
\end{aligned}
$$

(Use l'Hô pital's rule)

$$
\overset{l'H}{=} \lim_{x\to 0}\frac{-\ln(x+1) - 1}{\ln(x+1) + 1 + 1} = \frac{0 - 1}{0 + 1 + 1} = -\frac{1}{2}.
$$

$$
\lim_{x\to 0}\left(\frac{1}{\ln(x+1)} - \frac{x+1}{x}\right)^2 = \left[\lim_{x\to 0}\left(\frac{1}{\ln(x+1)} - \frac{x+1}{x}\right)\right]^2 = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}.
$$

   `Ans: B`

4.

(A)

$$\begin{cases} i=0, \ \frac{1}{2n}\frac{e^4}{3}=\frac{e^4}{6n}; \\ i=n, \ \frac{1}{2n}\frac{e^{10}}{6}=\frac{e^{10}}{12n}. \end{cases} \quad \begin{cases} \Delta x=\frac{5-2}{n}=\frac{3}{n}, \\ \Delta x\cdot f(2)=\frac{3}{n}\frac{e^4}{18}=\frac{e^4}{6n}, \\ \Delta x\cdot f(5)=\frac{3}{n}\frac{e^{10}}{36}=\frac{e^{10}}{12n}. \end{cases}$$

(B)

$$\begin{cases} i=0, \ \frac{1}{2n}\frac{e^4}{3}=\frac{e^4}{6n}; \\ i=n, \ \frac{1}{2n}\frac{e^{10}}{6}=\frac{e^{10}}{12n}. \end{cases} \quad \begin{cases} \Delta x=\frac{3-0}{n}=\frac{3}{n}, \\ \Delta x\cdot f(0)=\frac{3}{n}\frac{e^4}{6}=\frac{e^4}{2n}, \\ \Delta x\cdot f(3)=\frac{3}{n}\frac{e^{10}}{12}=\frac{e^{10}}{4n}. \end{cases}$$

(C)

$$\begin{cases} i=0, \ \frac{1}{2n}\frac{e^4}{3}=\frac{e^4}{6n}; \\ i=n, \ \frac{1}{2n}\frac{e^{10}}{6}=\frac{e^{10}}{12n}. \end{cases} \quad \begin{cases} \Delta x=\frac{1-0}{n}=\frac{1}{n}, \\ \Delta x\cdot f(0)=\frac{1}{n}\frac{e^4}{6}=\frac{e^4}{6n}, \\ \Delta x\cdot f(1)=\frac{1}{n}\frac{e^5}{8}=\frac{e^5}{8n}. \end{cases}$$

(D)

$$\begin{cases} i=0, \ \frac{1}{2n}\frac{e^4}{3}=\frac{e^4}{6n}; \\ i=n, \ \frac{1}{2n}\frac{e^{10}}{6}=\frac{e^{10}}{12n}. \end{cases} \quad \begin{cases} \Delta x=\frac{6-3}{n}=\frac{3}{n}, \\ \Delta x\cdot f(3)=\frac{3}{n}\frac{e^7}{18}=\frac{e^7}{6n}, \\ \Delta x\cdot f(6)=\frac{3}{n}\frac{e^{13}}{36}=\frac{e^{13}}{12n}. \end{cases}$$

`Ans: A`

5. Notice that e^{-x^2} is strictly decreasing on $[0,1]$. Thus

$$\int_0^{1/2} e^{-(1/2)^2}\,dx < \int_0^{1/2} e^{-x^2}\,dx < \int_0^{1/2} e^{-0^2}\,dx \Rightarrow \frac{1}{2}e^{-(1/4)} < \int_0^{1/2} e^{-x^2}\,dx < \frac{1}{2}.$$

Similarly, we have

$$\int_{1/2}^1 e^{-(1)^2}\,dx < \int_{1/2}^1 e^{-x^2}\,dx < \int_{1/2}^1 e^{-(1/2)^2}\,dx$$

$$\Rightarrow \frac{1}{2}e^{-1} < \int_{1/2}^1 e^{-x^2}\,dx < \frac{1}{2}e^{-(1/4)}.$$

Clearly.

$$L = \int_0^{1/2} e^{-x^2}\,dx + \int_{1/2}^1 e^{-x^2}\,dx.$$

In summary,

$$\frac{1}{2}\left(e^{-(1/4)}+e^{-1}\right) < L < \frac{1}{2}\left(1+e^{-(1/4)}\right) < 1.$$

`Ans: C`

6.

$$\int_0^1 \frac{x^2+x+2}{x^2+1}\ dx$$
$$= \int_0^1 1+\frac{x}{x^2+1}+\frac{1}{x^2+1}\ dx$$
$$= x+\frac{1}{2}\ln(x^2+1)+\tan^{-1}x\Big|_{x=0}^{1}$$
$$= 1+\frac{\ln 2}{2}+\frac{\pi}{4} = \frac{\pi+4+2\ln 2}{4}.$$

`Ans: C`

7.

$$S = \int 2\pi y\ ds = \int_0^3 2\pi y\sqrt{1+(y')^2}\ dx = \int_0^3 2\pi x^4\sqrt{1+(4x^3)^2}\ dx.$$

`Ans: C`

8. For $x \ge e$, $\dfrac{1+e^{-x^2}}{x} > \dfrac{1}{x} > 0$, and $\int_e^\infty \dfrac{1}{x}\ dx$ diverges.

By Comparison Theorem, $\int_e^\infty \dfrac{1+e^{-x^2}}{x}\ dx$ diverges.

`Ans: D`

9. The area of the region A is

$$\int y\ dx = \int_0^\pi y(\theta)x'(\theta)\ d\theta = \int_0^\pi y^2(\theta)\ d\theta$$
$$= \int_0^\pi r^2\left(1-\cos\theta\right)^2\ d\theta = r^2\int_0^\pi 1-2\cos\theta+\frac{\cos(2\theta)+1}{2}\ d\theta$$
$$= r^2\left(\frac{3}{2}\theta-2\sin\theta+\frac{1}{4}\sin(2\theta)\Big|_{\theta=0}^{\pi}\right) = \frac{3}{2}\pi r^2.$$

`Ans: B`

10. Let $r_1(\theta)=3$ and $r_2(\theta)=2+2\cos\theta$. Notice that $0\le r_2(\theta)\le 4$ for all $\theta\in\mathbb{R}$, and

$$(r_1(\theta_1)\cos\theta_1, r_1(\theta_1)\sin\theta_1) = (r_2(\theta_2)\cos\theta_2, r_2(\theta_2)\sin\theta_2)$$
$$\Leftrightarrow\ \theta_1=\theta_2 \text{ and } 2+2\cos\theta_2=3$$
$$\Leftrightarrow\ \theta_2 = 2n\pi\pm\frac{\pi}{3}\quad \forall\, n\in\mathbb{Z}.$$

Notice that while $0 \le \theta \le \frac{\pi}{6}$, $0 \le r_1(\theta) = 3 \le r_2(\theta) = 2 + 2\cos\theta$. Hence the **area** A is equal to

$$\begin{aligned}
&2\int_0^{\pi/3} \frac{1}{2}\left[{r_2}^2(\theta) - {r_1}^2(\theta)\right]\, d\theta = \int_0^{\pi/3} \left[4\cos^2\theta + 8\cos\theta - 5\right]\, d\theta \\
&= \int_0^{\pi/3} [2\cos(2\theta) + 8\cos\theta - 3]\, d\theta = -\pi + \left(\sin(2\theta) + 8\sin\theta|_{\theta=0}^{\pi/3}\right) \\
&= -\pi + \frac{\sqrt{3}}{2} + 8\cdot\frac{\sqrt{3}}{2} = \frac{9}{2}\sqrt{3} - \pi.
\end{aligned}$$

`Ans: C`

11.

(A)

$$f'(x) = \frac{\frac{2}{x}\cdot 2x^2 - \ln x^2 \cdot 4x}{4x^4} = \frac{1 - \ln x^2}{x^3} = 0$$

or dose not exist when $x = \pm\sqrt{e}$ (0 not in domain).
$f'(x) > 0$ and so f is increasing on $(-\infty, -\sqrt{e}) \cup (0, \sqrt{e})$.

(B)

$$f''(x) = \frac{-\frac{2}{x}\cdot x^3 - (1 - \ln x^2)\cdot 3x^2}{x^6} = \frac{3\ln x^2 - 5}{x^4}.$$

$f''(x) < 0$ and so is concave downward on $(-e^{5/6}, 0) \cup (0, e^{5/6})$.

(C)

$$f''(x) = \frac{-\frac{2}{x}\cdot x^3 - (1 - \ln x^2)\cdot 3x^2}{x^6} = \frac{3\ln x^2 - 5}{x^4},\ f''(x) = 0 \Leftrightarrow x = \pm e^{5/6}.$$

So f has two inflection points $\left(\pm e^{5/6}, \frac{5}{6}e^{-5/3}\right)$.

(D)

$$f'(x) = \frac{\frac{2}{x}\cdot 2x^2 - \ln x^2 \cdot 4x}{4x^4} = \frac{1 - \ln x^2}{x^3} = 0$$

or dose not exist when $x = \pm\sqrt{e}$ (0 not in domain).

$f'(x) > 0$ and so f is increasing on $(-\infty, -\sqrt{e}) \cup (0, \sqrt{e})$. $f'(x) < 0$ and so f is decreasing on $(-\sqrt{e}, 0) \cup (\sqrt{e}, \infty)$. So f has local maximum while $x = \pm\sqrt{e}$. Since $f(\pm\sqrt{e}) = \frac{1}{2e}$, the absolute maximum of f is $\frac{1}{2e}$.
`Ans: BD`

12.

(A) Consider $f(x) = x^3$ on $(-1, 1)$. Then $f'(0) = 3 \cdot 0^2 = 0$. But for any $t \in (0, 1)$,

$$f(t) = t^3 > f(0) = 0 > f(-t) = -t^3.$$

Therefore, $f(x)$ has no local extrema at $x = 0$.

(B) Consider $f(x) = x$ on $(-1, 1)$. Then f is continuous on $(-1, 1)$. But for any $c \in (-1, 1)$, $\ell := \dfrac{(-1) + c}{2}$, $s := \dfrac{1 + c}{2}$ are both belong to $(-1, 1)$, and

$$-1 < f(\ell) = \ell < f(c) = c < f(s) = s < 1.$$

Hence $f(c)$ is not an absolute extrema for any $c \in (-1, 1)$.

(C) If a function is differentiable on an open interval in $\mathbb{R}$, then it is continuous on this interval.

(D) Let $f(x) = |x|$ on $(-1, 1)$. Then $f^2(x) = x^2$ is differentiable on $(-1, 1)$. But f is not differentiable at 0.
`Ans: ABD`

13.

(A) Clearly, $\lim_{x \to 0^-} g(x) = 1$. Set $f(t) = \dfrac{1}{t^2 + \sqrt{t} + 1}$ for $t \geq 0$, and $F(x) = \int_0^x f(t)\, dt$ for $x \geq 0$. (Notice that $F(0) = 0$.) Hence

$$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} \frac{F(\sqrt{x})}{x}.$$

We will apply l'Hô pital's rule later. By fundamental theorem of calculus, for $x > 0$,

$$\frac{d}{dx} F(\sqrt{x}) = \frac{1}{2\sqrt{x}} f(\sqrt{x}) = \frac{1}{2\sqrt{x}} \frac{1}{x + \sqrt[4]{x} + 1}.$$

Now, by l'Hô pital's rule,

$$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} \frac{1}{2\sqrt{x}} \frac{1}{x + \sqrt[4]{x} + 1} = +\infty.$$

(B) $0 < \dfrac{1}{t^2 + \sqrt{t} + 1} < \dfrac{1}{t^2 + 1}$ for $t > 0$, so

$$0 \leq \lim_{x \to \infty} \frac{1}{x} \int_0^{\sqrt{x}} \frac{1}{t^2 + \sqrt{t} + 1}\, dt \leq \lim_{x \to \infty} \frac{1}{x} \int_0^{\sqrt{x}} \frac{1}{t^2 + 1}\, dt = \lim_{x \to \infty} \frac{1}{x} \tan^{-1} \sqrt{x} = 0 \cdot \frac{\pi}{2} = 0.$$

(C) It is obvious that $\lim_{x\to 0^-} \sin(x)g(x) = 0 \cdot 1 = 0$, and

$$\lim_{x\to 0^+} \sin(x)g(x) = \lim_{x\to 0^+} \left(\frac{\sin x}{x} \int_0^{\sqrt{x}} \frac{1}{t^2+\sqrt{t}+1}\, dt \right)$$

$$= \left(\lim_{x\to 0^+} \frac{\sin x}{x} \right) \cdot \int_0^{\sqrt{0}} \frac{1}{t^2+\sqrt{t}+1}\, dt = 1 \cdot 0 = 0.$$

(D) Set $f(t) = \dfrac{1}{t^2+\sqrt{t}+1}$ for $t \geq 0$, and $F(x) = \int_0^x f(t)\, dt$ for $x \geq 0$. (Notice that $F(0) = 0$.) By fundamental theorem of calculus, for $x > 0$,

$$\frac{d}{dx} F(\sqrt{x}) = \frac{1}{2\sqrt{x}} f(\sqrt{x}) = \frac{1}{2\sqrt{x}} \frac{1}{x + \sqrt[4]{x} + 1}.$$

Now,

$$L := \lim_{x\to 1} g'(x) = \lim_{x\to 1} \left(\frac{-1}{x^2} F(\sqrt{x}) + \frac{1}{x} \frac{1}{2\sqrt{x}} f(x) \right) = -F(1) + \frac{1}{6}.$$

Since $\dfrac{1}{t^2+\sqrt{t}+1} \geq \dfrac{1}{3}$ for $0 \leq t \leq 1$,

$$F(1) = \int_0^1 \frac{1}{t^2+\sqrt{t}+1}\, dt \geq \int_0^1 \frac{1}{3}\, dt = \frac{1}{3}.$$

Hence

$$L = -F(1) + \frac{1}{6} \leq -\frac{1}{3} + \frac{1}{6} = -\frac{1}{6} < 0.$$

`Ans: BC`

14.

(A) It is okay that f is continuous for $x \neq 0$. For $x = 0$, it is clearly that $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 0 = 0$. For the right-hand side, since

$$-x^2 \leq x^2 \sin\left(\frac{1}{x^3}\right) \leq x^2$$

and $\lim_{x\to 0} \pm x^2 = \pm 0^2 = 0$, by squeeze theorem,

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 \sin\left(\frac{1}{x^3}\right) = 0.$$

Hence $\lim_{x\to 0} f(x) = 0 = f(0)$, f is continuous on $\mathbb{R}$.

(B) It is okay that f is differentiable for $x \neq 0$. For $x = 0$,

$$\lim_{h\to 0^-} \frac{f(h)-f(0)}{h-0} = \lim_{h\to 0^-} 0 = 0. \lim_{h\to 0^+} \frac{f(h)-f(0)}{h-0} = \lim_{h\to 0^+} h\sin\left(\frac{1}{h^3}\right) = 0$$

by squeeze theorem. In summary,

$$f'(0) = \lim_{h\to 0} \frac{f(h)-0}{h-0} = 0,$$

so f is differentiable on $\mathbb{R}$.

(C)

$$\lim_{h\to 0^-} \frac{f(h)-f(0)}{h-0} = \lim_{h\to 0^-} 0 = 0. \lim_{h\to 0^+} \frac{f(h)-f(0)}{h-0} = \lim_{h\to 0^+} h\sin\left(\frac{1}{h^3}\right) = 0$$

by squeeze theorem. In summary,

$$f'(0) = \lim_{h\to 0} \frac{f(h)-0}{h-0} = 0.$$

(D)

$$f'(x) = 2x\sin\left(\frac{1}{x^3}\right) + x^2\cos\left(\frac{1}{x^3}\right)\cdot\left(\frac{-3}{x^4}\right) = 2x\sin\left(\frac{1}{x^3}\right) - \left(\frac{3}{x^2}\right)\cos\left(\frac{1}{x^3}\right),$$

thus the limit $\lim_{x\to 0^+} f'(x)$ does not exist, f' is not continuous at 0.

`Ans: ABC`

15.

(I)

$$\begin{aligned}\int_0^1 e^{\sqrt{x}}\,dx &= \int_0^1 e^u\,d(u^2) = \int_0^1 2ue^u\,du \\ &= \int_0^1 2u\,d(e^u) = 2ue^u\Big|_0^1 - \int_0^1 e^u\,d(2u) \\ &= \Big[2ue^u - 2e^u\Big]_0^1 = 2.\end{aligned}$$

(II)

$$\int_0^1 xe^x\,dx = \Big[xe^x - e^x\Big]_0^1 = 1.$$

(III)

$$\begin{aligned}&\int_0^{\pi/2} e^{\sin x}\sin 2x\,dx \\ =&\int_0^{\pi/2} 2e^{\sin x}\sin x\cos x\,dx \\ =&\int_0^1 2ve^v\,dv = 2.\end{aligned}$$

`Ans:` C

16. Let $y = \left(\frac{2^x + 5^x}{2}\right)^{\frac{1}{x}}$.

$$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{\ln\left(\frac{2^x+5^x}{2}\right)}{x} = \lim_{x\to 0} \frac{\left(\frac{(\ln 2)2^x + (\ln 5)5^x}{2}\right)}{\left(\frac{2^x+5^x}{2}\right)\cdot 1}$$

$$= \frac{(\ln 2 + \ln 5)/2}{(1+1)/2} = \frac{\ln 2 + \ln 5}{2} = \ln\sqrt{10}.$$

Thus $\lim_{x\to 0} y = \exp(\lim_{x\to 0} \ln y) = \sqrt{10}$.
`Ans:` $\sqrt{10}$

17. For change of variables, set $u = x^2 + x + 1$. Then $du = (2x+1)\,dx$, and

$$I_1 := \int \frac{x + \frac{1}{2}}{\sqrt{x^2+x+1}}\,dx = \int \frac{\frac{1}{2}\,du}{\sqrt{u}} = \sqrt{u} + C_1 = \sqrt{x^2+x+1} + C_1.$$

For the rest part $I_2 := -\frac{1}{2}\int \frac{1}{\sqrt{x^2+x+1}}\,dx$, we observe that

$$x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 \left[\left(\frac{2x}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)^2 + 1\right].$$

By change of variables,
set $\sinh v = \frac{2x}{\sqrt{3}} + \frac{1}{\sqrt{3}}$, then $\sqrt{x^2+x+1} = \frac{\sqrt{3}}{2}\sqrt{\sinh^2 v + 1} = \frac{\sqrt{3}}{2}\cosh v$,
$\cosh v\,dv = \frac{2}{\sqrt{3}}\,dx$,
and

$$I_2 = -\frac{1}{2}\int \frac{2}{\sqrt{3}}\frac{1}{\cosh v}(\cosh v)\cdot\frac{\sqrt{3}}{2}\,dv$$

$$= -\frac{1}{2}\int dv = -\frac{1}{2}v + C_2 = -\frac{1}{2}\sinh^{-1}\left(\frac{2x}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right) + C_2.$$

Since $\sinh^{-1} y = \ln\left(y + \sqrt{1+y^2}\right)$, we have that

$$I_2 = -\frac{1}{2}\ln\left(\frac{2x}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}\sqrt{x^2+x+1}\right) + C_2$$

$$= -\frac{1}{2}\ln\left(\sqrt{x^2+x+1} + x + \frac{1}{2}\right) - \frac{1}{2}\ln\frac{2}{\sqrt{3}} + C_2.$$

In summary,

$$\int \frac{x}{\sqrt{x^2+x+1}}\,dx = I_1 + I_2 = \sqrt{x^2+x+1} - \frac{1}{2}\ln\left(\sqrt{x^2+x+1} + x + \frac{1}{2}\right) + C.$$

`Ans:` $\sqrt{x^2+x+1} - \frac{1}{2}\ln\left(\sqrt{x^2+x+1} + x + \dfrac{1}{2}\right) + C$

18. $x^2 + y^2 = r^2$, $A(x) = \dfrac{y^2}{2}$. So the volume is

$$V = \int A(x)\,dx = \int_{-r}^{r} \frac{y^2}{2}\,dx = \int_0^r r^2 - x^2\,dx = r^2x - \frac{x^3}{3}\bigg|_{x=0}^{r} = \frac{2}{3}r^3 = 144.$$

`Ans:` `144`

19. By fundamental theorem of calculus, $\dfrac{dy}{dx} = \sqrt{(\ln x)^2 - 1}$

The length of the curve is $L =$

$$\int_e^{e^2} \sqrt{1+(y')^2}\,dx = \int_e^{e^2} \sqrt{1+(\ln x)^2 - 1}\,dx = \int_e^{e^2} |\ln x|\,dx = x\ln x - x|_{x=e}^{e^2} = e^2.$$

`Ans:` e^2

20. The length of the polar curve L is

$$\int_0^{\pi} \sqrt{r^2 + (r')^2}\ d\theta = \int_0^{\pi} \sqrt{(1+\sin\theta)^2 + (\cos\theta)^2}\ d\theta$$

$$= \sqrt{2}\int_0^{\pi} \sqrt{2+2\sin\theta}\ d\theta = 2\sqrt{2}\int_0^{\pi/2} \sqrt{1+\sin\theta}\ d\theta$$

$$(\cos\left(\frac{\pi}{2}+t\right) = \cos\left(\frac{\pi}{2}-t\right) \text{ for } t \in \mathbb{R}.)$$

$$= 2\sqrt{2}\int_0^{\pi/2} \frac{\sqrt{1+\sin\theta}\sqrt{1-\sin\theta}}{\sqrt{1-\sin\theta}}\ d\theta = 2\sqrt{2}\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{1-\sin\theta}}\ d\theta$$

For change of variables, let $u = \sin\theta$. Then $du = \cos\theta\ d\theta$. Thus

$$L = 2\sqrt{2}\int_0^1 \frac{1}{\sqrt{1-u}}\ du = -4\sqrt{2}\sqrt{1-u}\bigg|_{u=0}^{1} = 4\sqrt{2}.$$

`Ans:` $4\sqrt{2}$